Solutions to Homework Assignment 28

MATH 345-01

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6. $u(x,y) = \Re z = e^x \cos y$. Thus $u_x = e^x \cos y$ and $u_y = -e^x \sin y$. These are never simultaneously zero, so u has no critical points. On the boundary of the region, we have four curves to consider.

   (a) $x = 0 : u(x,y) = \cos y$, which has a max of 1 at $y = 0$ and a min of -1 at $y = \pi$.

   (b) $x = 1 : u(x,y) = e \cos y$, which has a max of e at $y = 0$ and a min of $-e$ at $y = \pi$.

   (c) $y = 0 : u(x,y) = e^x$, which has a max of e at $x = 1$ and a min of 1 at $x = 0$.

   (d) $y = \pi : u(x,y) = -e^x$, which has a max of -1 at $x = 0$ and a min of $-e$ at $x = 1$.

   Thus the overall max is e at $(1,0) = 1$ and the overall min is $-e$ at $(1,\pi) = 1 + i\pi$.