Solutions to Homework Assignment 29

MATH 345

Section 61, Page 185

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1. Let $\epsilon > 0$ be given, and let $N = \lceil\sqrt{1/\epsilon}\rceil$. Then for $n > N$, we have $|(1/n^2 + i) - i| = |1/n^2| < 1/N^2 \leq \epsilon$, as desired.

6. Suppose that $\sum_{n=1}^{\infty} z_n = S = X + iY$. If $z_n = x_n + iy_n$, then we know by the theorem in the section that

$$\begin{aligned}\sum_{n=1}^{\infty} \overline{z_n} &= \sum_{n=1}^{\infty}(x_n - iy_n)\\ &= \sum_{n=1}^{\infty} x_n - i\sum_{n=1}^{\infty} y_n\\ &= X - iY\\ &= \overline{S},\end{aligned}$$

as desired.

7. Suppose that $\sum_{n=1}^{\infty} z_n = S = X + iY$. If $z_n = x_n + iy_n$ and $c = a + bi$, then we know by the theorem in the section that

$$\begin{aligned}\sum_{n=1}^{\infty} \overline{cz_n} &= \sum_{n=1}^{\infty}(ax_n - by_n) + i(bx_n + ay_n)\\ &= \sum_{n=1}^{\infty}(ax_n - by_n) + i\sum_{n=1}^{\infty}(bx_n + ay_n)\\ &= aX - bY + i(bX + aY)\\ &= (a + bi)(X + iY)\\ &= cS,\end{aligned}$$

as desired.

8. With $S = X + iY, z_n = x_n + iy_n, T = U + iV$, and $w_n = u + iv_n$, we have

$$\begin{aligned}\sum_{n=1}^{\infty} \overline{z_n + w_n} &= \sum_{n=1}^{\infty}(x_n + iy_n + u_n + iv_n)\\ &= \sum_{n=1}^{\infty}(x_n + u_n) + i\sum_{n=1}^{\infty}(y_n + v_n)\\ &= X + U + i(Y + V)\\ &= (X + iY) + (U + iV)\\ &= S + T,\end{aligned}$$

as desired.