Solutions to Homework Assignment 30

MATH 345

Section 65, Page 195

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1. Since $\cosh z = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}$, $z\cosh(z^2) = z\sum_{n=0}^{\infty} \frac{(z^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!}$.

2. (a) Since $f^{(n)}(z) = e^z$, $f^{(n)}(1) = e$ for all n. Thus the series is $\sum_{n=0}^{\infty} e\frac{(z-1)^n}{n!}$.

   (b) We have $e^z = e^{z-1}e = e\sum_{n=1}^{\infty} \frac{(z-1)^n}{n!}$.

8. (a)

$$\begin{aligned}
\cos z &= \frac{e^{iz} + e^{-iz}}{2} \\
&= \frac{1}{2}\sum_{n=0}^{\infty} \frac{i^n z^n + (-1)^n i^n z^n}{n!} \\
&= \frac{1}{2}\sum_{n=0}^{\infty} \frac{i^{2n} z^{2n}}{(2n)!} \\
&= \frac{1}{2}\sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!},
\end{aligned}$$

   as desired.

   (b) We did this in class.