Solutions to Homework Assignment 32

MATH 345

Section 72, Page 218

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1. $\frac{d}{dz}(1-z)^{-1} = \frac{1}{(1-z)^2}$, and since $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$, we also have $\frac{d}{dz}\frac{1}{1-z} = \sum_{n=1}^{\infty} nz^{n-1}$.

   Reindexing to start at $n = 0$ makes this $\sum_{n=0}^{\infty}(n+1)z^n$.

   Differentiating again gives $\frac{d}{dz}\frac{1}{(1-z)^2} = \frac{2}{(1-z)^3} = \sum_{n=1}^{\infty} n(n+1)z^{n-1} = \sum_{n=0}^{\infty}(n+1)(n+2)z^n$.

2. Following instructions, we have

$$\frac{1}{\left(1-\frac{1}{1-z}\right)^2} = \sum_{n=0}^{\infty}(n+1)\left(1-\frac{1}{1-z}\right)^n$$

$$\frac{(1-z)^2}{z^2} = \sum_{n=0}^{\infty}(n+1)\left(\frac{1}{1-z}\right)^n$$

$$\frac{1}{z^2} = \sum_{n=0}^{\infty}(n+1)\frac{1}{(1-z)^{n+2}}$$

$$\frac{1}{z^2} = \sum_{n=2}^{\infty}(n-1)\left(\frac{1}{1-z}\right)^n$$

$$\frac{1}{z^2} = \sum_{n=0}^{\infty}\frac{(-1)^n(n-1)}{(z-1)^n}.$$

   Since we need $|z| < 1$ for convergence of the original series, we need $\frac{1}{|z-1|} < 1$ now, which means $|z-1| > 1$.