

MATH 356-01

Review for Final

NOTE: This list may not be exhaustive. You are responsible for all material to date.

This exam is out of 130 points. Roughly 2/3 of the points will be computational or conceptual and 1/3 proof-based.

1. Major definitions and theorems.

- (a) Linear Diophantine equations, including finding all solutions.
- (b) Algebraic structures: groups, rings, fields, integral domains, Euclidean domains, and UFDs.
- (c) The Euclidean Algorithm (finding gcds) and Bezout's Identity (in all of our various contexts).
- (d) $\mathbb{Z}, \mathbb{Z}[i], \mathbb{Z}[\zeta_3], \mathbb{Z}[\zeta_n], \mathbb{Z}[\sqrt{d}], \mathbb{H}$.
- (e) The Fundamental Theorem of Arithmetic and the Fundamental Meta-Theorem of Arithmetic.
- (f) Euclid's Lemma and the Prime Divisor Lemma.
- (g) Wilson's Theorem
- (h) Fermat's Little Theorem
- (i) Euler's Theorem
- (j) Lagrange's Theorem
- (k) Lagrange's Four-Square Theorem
- (l) Sun Tzu's Theorem
- (m) The Path (to unique factorization)
- (n) Algebraic numbers and algebraic integers
- (o) The Law of Quadratic Reciprocity (and connections to equations of the form $p = x^2 - dy^2$)
- (p) The Prime Number Theorem
- (q) τ, σ, μ

Note that I have not listed everything here. These are just some of the major named results; most of them rely on knowing underlying ideas, as well.

2. Logistics.

- (a) Tuesday, December 11, 8-11am in Ford 201.
- (b) One side of one 8.5" \times 11" page of notes.
- (c) Calculator okay (no cell phone calculators). **Python?**
- (d) Problems on one page. I will provide paper for your work.