

MATH 356-01

Solutions to Homework Assignment 2

- 3.2 (a) $5 + 7i = (2 + 3i)(a + bi) \implies 5 + 7i = (2a - 3b) + (2b + 3a)i$. Thus we need $2a - 3b = 5$ and $2b + 3a = 7$. Since the solutions to this aren't integers, $2 + 3i \nmid 5 + 7i$. Alternatively (and smarterly), $N(2 + 3i) = 13$, while $N(5 + 7i) = 74$, and $13 \nmid 74$.
- (b) $5 + 0i = (3 + 0i)(a + bi) \implies 5 = 3a$. Thus $3 \nmid 5$. $N(3) = 9 \nmid 25 = N(5)$.
- (c) Here, $N(2 + i) = 5 \mid 50 = N(7 + i)$, so we have to do the work. $7 + i = (2 + i)(a + bi) \implies 7 + i = (2a - b) + (2b + a)i$. Thus we need $2a - b = 7$ and $2b + a = 1$. $a = 3, b = -1$ is a solution, so $2 + i \mid 7 + i$.
- (d) Here, $10 = 2(2 + i)(2 - i)$, so $2 + i \mid 10$.
- 3.5 $\gcd(G - 1, G + 1) = \gcd(G - 1, (G + 1) - (G - 1)) = \gcd(G - 1, 2) = 1$ since $G - 1$ is odd.
- 3.17 If $a \mid b$, then there exists $m \in \mathbb{Z}$ such that $b = am$. If $b \mid c$, then there exists $n \in \mathbb{Z}$ such that $c = bn$. Thus $c = bn = (am)n = a(mn)$, so $a \mid c$.
- 3.20 (a) If u and v are units, then uv is a unit since $(v^{-1}u^{-1})(uv) = v^{-1}(u^{-1}u)v = v^{-1}1v = v^{-1}v = 1$ and $(uv)(v^{-1}u^{-1}) = u(vv^{-1})u^{-1} = u1u^{-1} = uu^{-1} = 1$, so uv has an inverse.
- (b) If u is a unit, so is u^{-1} since the inverse of u^{-1} exists (it's u , of course!).
- (c) If u is a unit, so is $-u$ since $(-(u^{-1}))(-u) = u^{-1}u = 1$ and $(-u)(-(u^{-1})) = uu^{-1} = 1$. Thus, $(-u)^{-1} = -(u^{-1})$ (so we just go ahead and write $-u^{-1}$).