MATH 356-01
Solutions to Homework Assignment 3

3.7 Since 20! is made up of factors of all of the integers up to 20, its prime factorization only contains those primes less than 20: 2, 3, 5, 7, 11, 13, 17, and 19. 11, 13, 17, and 19 each occur only once since even $2 \cdot 11 > 20$. There are $20/2 = 10$ multiples of 2 included, $20/4 = 5$ multiples of 4 (each contributing one more factor of 2), 20/8 rounded down (2) factors of 8 (each also contributing one more factor of 2), and a factor 16, which gives one more factor of 2 we haven't yet accounted for. All together, we have $10 + 5 + 2 + 1 = 18$ factors of 2 in 20!. Similarly, there are $6 + 2 = 8$ factors of 3, 4 factors of 5, and 2 factors of 7. We see that the prime factorization of 20! is $2^{18} \cdot 3^8 \cdot 5^4 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19$.

3.12 The problem with $p = 6$ is that $6|a^2$ does not imply $6|a$. However, $6 = 2 \cdot 3$, so we could argue separately for 2 and 3 to find that, in this situation, $2|a$ and $3|a$, so $6|a$.

3.13a One solution is $x = -1, y = 1$. Since $\gcd(24, 40) = 8$, all other solutions have the form $x = -1 + \frac{40}{8}t = -1 + 5t, y = 1 - \frac{24}{8}t = 1 - 3t$.

3.14 The only possible rational roots are the divisors of 8: $\pm 1, \pm 2, \pm 4, \pm 8$. We see that $x = 1$ is the only rational root.