

MATH 356-01

Solutions to Homework Assignment 4

- 4.7 (a) $1024 \cdot 3728 + 1111^2 \equiv 4 \cdot 8 + 1^2 \equiv 3 \pmod{10}$.
(b) $2^{10000} \equiv (-1)^{10000} \equiv 1 \pmod{3}$
(c) $3^{42} \equiv 2^{21} \equiv 1^7 \equiv 1 \pmod{7}$ (since $2^3 \equiv 1 \pmod{7}$).
(d) $\sum_{n=1}^{100} n! \equiv 1! + 2! + 3! + 0 + \dots + 0 \equiv 9 \pmod{12}$ (since $n! \equiv 0 \pmod{12}$ for $n \geq 4$).

4.9 Following the hint, we have $512 \cdot 19?3125 \equiv -(3+?) \pmod{9}$, while $100000000 \equiv 1 \pmod{9}$. Thus $-? \equiv 4 \pmod{9}$, so $? = 5$.

4.11

$$77 = 52(1) + 25$$

$$52 = 25(2) + 2$$

$$25 = 2(12) + \boxed{1}$$

$$2 = 1(2) + 0.$$

Working backwards, we now find

$$1 = 25 + 2(-12)$$

$$= 25 + (52 + 25(-2))(-12) = 52(-12) + 25(25)$$

$$= 52(-12) + (77 - 52(1))(25) = 77(25) + 52(-37).$$

Since $-37 \equiv 40 \pmod{77}$, $52^{-1} \equiv 40 \pmod{77}$. This gives $52x \equiv 3 \pmod{77} \implies 40(52x) \equiv 40(3) \equiv 43 \pmod{77}$, or $x \equiv 43 \pmod{77}$.