MATH 356-01
Solutions to Homework Assignment 10

6.14 Norm to the rescue! $N(3-2\zeta_3) = 3^2 + (-2)^2 - (3)(-2) = 19$ is prime, so $3-2\zeta_3$ is prime, too.

6.15

$$\begin{aligned}\frac{5-3\zeta_3}{3+2\zeta_3} &= \frac{5-3\zeta_3}{3+2\zeta_3}\cdot\frac{3+2\zeta_3^2}{3+2\zeta_3^2}\\ &= \frac{15+10(-1-\zeta_3)-9\zeta_3-6}{7}\\ &= -\frac{1}{7}-\frac{19}{7}\zeta_3.\end{aligned}$$

I will round to $0-3\zeta_3 = -3\zeta_3$. Now

$$\begin{aligned}5-3\zeta_3-(-3\zeta_3)(3+2\zeta_3) &= 5-3\zeta_3+9\zeta_3+6(-1-\zeta_3)\\ &= -1.\end{aligned}$$

Thus $5-3\zeta_3 = (-3\zeta_3)(3+2\zeta_3)-1$.

Notice that each time ζ_3^2 would have appeared, I replaced it with $-1-\zeta_3$.