

# MATH 356 Number Theory

## Python Worksheet 3: Modular Stuff

1. Compute the order of each element in  $\mathbb{Z}/13^\times$ .
2. The following is an implementation of  $\varphi(n)$ , the function giving the number of integers from 1 to  $n$  that are relatively prime to  $n$ .

```
def phi(n):  
    relPrime=0  
    for i in range(1,n+1):  
        if gcd(i,n)==1:  
            relPrime+=1  
    return relPrime
```

(Refer back to a prior worksheet for our gcd function.)

Use your  $\varphi$  function to formulate a conjecture for  $\varphi(p^2)$ ,  $\varphi(p^3)$ ,  $\varphi(p^k)$ , and  $\varphi(pq)$  for distinct primes  $p$  and  $q$ . (This is essentially problem 4 from Worksheet H.)

3. For each unit  $a \in \mathbb{Z}/(7)$ , find the least positive exponent  $e$  such that  $a^e = 1$ . Now try it in  $\mathbb{Z}/(17)$ .
4. Find an exponent  $e$  such that  $a^e = 1$  for *all* nonzero  $a$  in  $\mathbb{Z}/(13)$ . (Same exponent!) Now try it in  $\mathbb{Z}/(17)$ .
5. Make a conjecture based on the last two problems.