Solutions to Homework −2

Section 1.1
p. 8: 1ace, 2, 3ace, 7

1. See back of book.

3. See back of book.

10. Suppose that a and c leave the same remainder when divided by n. Then there exist q_1, q_2, and r such that $a = nq_1 + r$ and $c = nq_2 + r$. Therefore,

$$\begin{aligned} a - c &= (nq_1 + r) - (nq_2 + r) \\ &= nq_1 - nq_2 \\ &= n(q_1 - q_2) \\ &= nk, \end{aligned}$$

as desired.

Now suppose that $a - c = nk$ for some integer k. Write $a = nq_1 + r_1$ and $c = nq_2 + r_2$, where $0 \le r_1, r_1 < n$. Without loss of generality we may assume that $r_1 \ge r_2$. Then

$$\begin{aligned} a - c &= (nq_1 + r_1) - (nq_2 + r_2) \\ &= n(q_1 - q_2) + (r_1 - r_2) \\ &= nk. \end{aligned}$$

This implies that $n(q_1 - q_2 - k) = r_1 - r_2$. Since $0 \le r_1 - r_2 < n$, we must have $0 \le n(q_1 - q_2 - k) < n$. The only multiple of n that satisfies this is 0, so $r_1 = r_2$.