MATH 456-01

Solutions to Homework -1

Section 1.2
p. 14: 1aceg, 2, 5, 8, 9

1. I use the Euclidean algorithm.

   (a)

   $$\begin{aligned} 72 &= 56 \cdot 1 + 16 \\ 56 &= 16 \cdot 3 + 8 \\ 16 &= 8 \cdot 2 + 0. \end{aligned}$$

   Thus, $(56, 72) = 8$.

2. Suppose first that $b|a$. Then $a = bn$ for some integer n. Therefore, $a = (-b)(-n)$, so $-b|a$. Now suppose that $-b|a$. By the forward direction, $-(-b)|a$ as well, so $b|a$.

5. Since $a|b$ and $b|a$, $b = am$ and $a = bn$ for some integers m and n, and neither a nor b is zero. Thus $b = am = (bn)m$, so $mn = 1$. Therefore, $m = n = 1$ or $m = n = -1$, and $b = \pm a$.

8. We have

   $$\begin{aligned} n + 1 &= n \cdot 1 + 1 \\ n &= 1 \cdot n + 0. \end{aligned}$$

   Therefore, $(n, n+1) = 1$.

9. Let $a = 3, b = 6$, and $c = 12$. Then $a|c$ and $b|c$, but $ab \not| c$. Suppose that $(a, b) = 1$. Write $c = am = bn$. Then $a|bn$. Since $(a, b) = 1$, we have $a|n$ by Theorem 1.5. Thus $n = aq$ for some integer q, so $c = bn = (ba)q$, so $ab|c$.