

MATH 456-01

Solutions to Homework 0

Section 1.3

p. 22: 1ac, 15, 16, 21

1. See back of book.
15. If $p|a^n$, then p divides a . (Apply induction and Theorem 1.8.) Thus $p^n|a^n$.
16. Suppose that $(a, b) = 1$. Let p be a prime dividing a . If $p|b$ as well, then $(a, b) \geq p$, a contradiction. Thus $p \nmid b$. Now assume that there is no prime p such that $p|a$ and $p|b$. Let $d = (a, b)$. If $d > 1$, then there exists a prime p such that $p|d$ by the FTA. But then $p|d$ and $d|a$, so $p|a$, and, similarly, $p|b$. This is a contradiction, so $(a, b) = 1$.
21. Let $c = p_1^{e_1} \cdots p_k^{e_k}$ be the prime factorization of c . Then $c^2 = p_1^{2e_1} \cdots p_k^{2e_k} = ab$. Since $(a, b) = 1$, if some $p_i|a$, then $p_i \nmid b$ by Exercise 8. Thus the primes p_1, \dots, p_k are partitioned into two sets: those dividing a (without loss of generality, p_1, \dots, p_r) and those dividing b (p_{r+1}, \dots, p_k). Thus $a = p_1^{2e_1} \cdots p_r^{2e_r} = (p_1^{e_1} \cdots p_r^{e_r})^2$ (a perfect square) and $b = p_{r+1}^{2e_{r+1}} \cdots p_k^{2e_k} = (p_{r+1}^{e_{r+1}} \cdots p_k^{e_k})^2$ (also a perfect square).