MATH 456-01

Solutions to Homework 2

Section 2.2
p. 36: 1ac, 3, 4, 5, 9, 16

1. (c)

$+$	0	1
0	0	1
1	1	0

,

$\cdot$	0	1
0	0	0
1	0	1

   (c) See back of book.

3. In $\mathbb{Z}_8, 0^2, 2^2, 4^2, 6^2 \neq 1$, but $1^2 = 3^2 = 5^2 = 7^2 = 1$.

4. In $\mathbb{Z}_5, 0^4 \neq 1$, but $1^4 = 2^4 = 3^4 = 4^4 = 1$.

5. In $\mathbb{Z}_6$, $0^2 + 3 \cdot 0 + 2 = 2, 1^2 + 3 \cdot 1 + 2 = 0, 2^2 + 3 \cdot 2 + 2 = 0, 3^2 + 3 \cdot 3 + 2 = 2, 4^2 + 3 \cdot 4 + 2 = 0$, and $5^2 + 3 \cdot 5 + 2 = 0$. Thus $1, 2, 4$, and 5 are all solutions.

9. (a) Certainly 0 and 1 can't work. Consider 2: $2^1 = 2, 2^2 = 4, 2^3 = 1, 2^4 = 2, \ldots$ and now we're just going to cycle. Try 3: $3^1 = 3, 3^2 = 2, 3^3 = 6, 3^4 = 4, 3^5 = 5, 3^6 = 1$. That works!

   (b) Try 2: $2^1 = 2, 2^2 = 4, 2^3 = 3, 2^4 = 1$. That works, too!

   (c) Try 2: $2^1 = 2, 2^2 = 4, 2^3 = 2, \ldots$. Try 3: $3^1 = 3, 3^2 = 3, \ldots$. Try 4: $4^1 = 4, 4^2 = 4, \ldots$. Try 5: $5^1 = 5, 5^2 = 1, 5^3 = 5, \ldots$. Nothing works!

16. 0 is not such a number for any $\mathbb{Z}_n$ since $0x = 0$.

   (a) $1 \cdot 1 = 1, 2 \cdot 3 = 1, 4 \cdot 4 = 1$, so everything except 0 will work.

   (b) $1 \cdot 1 = 1$ and $3 \cdot 3 = 1$, but $2 \cdot 0 = 0, 2 \cdot 1 = 2, 2 \cdot 2 = 0$, and $2 \cdot 3 = 2$. Thus, only 1 and 3 work.

   (c) $1 \cdot 1 = 1, 2 \cdot 2 = 1$.

   (d) $1 \cdot 1 = 1$ and $5 \cdot 5 = 1$. However, from the multiplication tables on page 33, we see that these are the only two.