MATH 456-01

Solutions to Homework 3

Section 2.3
p. 39: 1, 4, 6

1. (a) Since 7 is prime, a solution exists for all $a \neq 0$ in $\mathbb{Z}_7$.
   (b) From Theorem 2.11, we know that we have a solution if $(a, 8)|1$. That is, a must be relatively prime to 8. The a for which solutions exist are $1, 3, 5$, and 7.
   (c) Reasoning as in (b), the a for which solutions exist are $1, 2, 4, 5, 7$, and 8.
   (d) Again reasoning as in (b), we get $1, 3, 7$, and 9.

4. See exercises 12-14 for the "quick way" to think about this problem. Since the moduli are small, you can also just go through the options.
   (a) Since $(6, 7) = 1$ and $1|4$, there is 1 solution.
   (b) Since $(6, 8) = 2$ and $2|4$, there are 2 solutions.
   (c) Since $(6, 9) = 3$ and $3 \not| 4$, there are no solutions.
   (d) Since $(6, 10) = 2$ and $2|4$, there are 2 solutions.

6. Write $n = ab$ with $1 < a, b < n$. Then in $\mathbb{Z}_n$, $ab = n = 0$, while $a, b \neq 0$.