MATH 456-01

Solutions to Homework 7

Section 4.1
p. 93: 1, 3, 5ad, 8, 9

1. (a) See back of book.
   (b) In $\mathbb{Z}_3[x]$, $(x+1)^3 = x^3+3x^2+3x+1 = x^3+1$.
   (c) See back of book.
   (d) $(x^2-3x+2)(2x^3-4x+1) = 8x^5-6x^4+13x^2-11x+2 = x^5+x^4+6x^2+3x+2$.

3. (a) A polynomial of degree 3 has the form ax^3+bx^2+cx+d. In $\mathbb{Z}_2$, $a,b,c,d \in \{0,1\}$. In fact, $a=1$ since the degree is three. Therefore, there are 8 such polynomials: $x^3, x^3+1, x^3+x, x^3+x^2, x^3+x+1, x^3+x^2+1, x^3+x^2+x$, and x^3+x^2+x+1.
   (b) A polynomial of the desired kind has the form ax^2+bx+c, where $a,b,c \in \{0,1,2\}$. Thus, the polynomials are $0,1,2$ (degree ≤ 0), $x, x+1, x+2, 2x, 2x+1, 2x+2$ (degree 1), $x^2, 2x^2, x^2+1, x^2+2, x^2+x, x^2+2x, 2x^2+1, 2x^2+2, 2x^2+x, 2x^2+2x, x^2+x+1, x^2+x+2, x^2+2x+1, x^2+2x+2, 2x^2+x+1, 2x^2+x+2, 2x^2+2x+1$, and $2x^2+2x+2$ (degree 2).

5. $3x^4-2x^3+6x^2-x+2-3x^2(x^2+x+1) = -5x^3+3x^2-x+2$. $-5x^3+3x^2-x+2+5x(x^2+x+1) = 8x^2+4x+2$. $8x^2+4x+2-8(x^2+x+1) = -4x-6$. Thus $3x^4-2x^3+6x^2-x+2 = (3x^2-5x+8)(x^2+x+1)-4x-6$, so $q(x) = 3x^2-5x+8$ and $r(x) = -4x-6$.

8. Let $p(x) \in R[x]$. Then there exist $n \in \mathbb{Z}_{\geq 0}, a_0, a_1, \ldots, a_n \in R$ such that $p(x) = a_0 + a_1x+\ldots+a_nx^n$. Now $1_Rp(x) = 1_R(a_0+a_1x+\ldots+a_nx^n) = c_0+c_1x+c_2x^2+\ldots+c_{n+0}x^{n+0}$, where $c_i = 1_Ra_i + 0a_{i-1} + \ldots + 0a_1 + 0a_0 = a_i$. Therefore, $1_Rp(x) = p(x)$. Similarly, $p(x)1_R = p(x)$.

9. Yes; c is still a zero divisor in $R[x]$: since c is a zero divisor in R, there exists $d \neq 0$ in R such that $cd = 0_R$. But $R \subseteq R[x]$, so $d \in R[x]$, too, and $cd = 0_R = 0_{R[x]}$.