MATH 456-01

Solutions to Homework 13

Section 5.1
p. 129: 1, 3, 6, 10, 13

1. (a) $f(x)-g(x) = x^5-5x^4+2x^3+5x^2-3x-1 = (x^2+1)(x^3-5x^2+x+10)-4x-11$. $f(x)$ and $g(x)$ are not congruent mod $p(x)$. (We could also have checked to see whether i is a root. It isn't.)

   (b) $f(x)-g(x) = x^5-2x^4+4x^3-8x^2+3x-2$ looks like a nuisance to divide, but notice that 1 is a root of $p(x)$ and not a root of $f(x)-g(x)$. Therefore, $f(x)$ is not congruent to $g(x)$ mod $p(x)$.

3. According to Corollary 5.5, the set of polynomials of degree at most 2 in $\mathbb{Z}_2[x]$ gives a complete set of representatives of the congruence classes mod x^3+2x+1. These have the form ax^2+bx+c. Since there are two choices for each coefficient and there are three coefficients, there are $2^3 = 8$ such polynomials.

6. Since the only polynomials of degree less than 1 are constant polynomials, constants give a complete set of congruence classes modulo $x-a$.

10. This is true: Suppose $p(x)$ is irreducible in $F[x]$ and $f(x)g(x) \equiv 0_F \pmod{p(x)}$. Then $p(x)|f(x)g(x)$, and, since $p(x)$ is irreducible, we have that $p(x)|f(x)$ or $p(x)|g(x)$ by Theorem 4.11. Thus $f(x) \equiv 0_F \pmod{p(x)}$ or $g(x) \equiv 0_F \pmod{p(x)}$.

13. If $x|f(x)-g(x)$, then $f(x)$ and $g(x)$ have the same constant term, so their graphs have the same y-intercept.