

Strategy (Groups of order 8)

Theorem

If G is a group of order 8, then $G \cong \mathbb{Z}_8, \mathbb{Z}_4 \times \mathbb{Z}_2, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2, D_4$, or Q (the quaternions).

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