

## Strategy ( $A_n$ is simple)

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3. Three cases for cycles appearing in elements of  $N$ :
4. Case 1: Some cycle has length 4. Construct a 3-cycle in  $N$ .
5. Case 2: Some cycle has length 3 but none are longer. Construct a 5-cycle or a 3-cycle in  $N$ .

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