MATH 457, Abstract Algebra II
Brief Summary of 456

	Rings	Groups
Setup	Set R, two bin ops $+, \cdot$	Set G, one bin op $\circ$
Operations	$+$: assoce, comm, ID, inverses $\cdot$: assoc $\cdot$ dist over $+$	$\circ$: assoc, ID, inverses
Special subsets	Subring S: $a, b \in S \implies ab, a-b \in S$ Ideal $I : a, b \in I \implies a - b \in S$, and $a \in S, b \in R \implies ab, ba \in S$	Subgroup H: $a, b \in H \implies ab^{-1} \in H$ Normal subgroup H: $a \in G, b \in H \implies aba^{-1} \in H$
Products	$R_1 \times R_2 \times \cdots \times R_k$	$G_1 \times G_2 \times \cdots \times G_k$ or $G_1 \oplus G_2 \oplus \cdots \oplus G_k$
Quotients	R/I a ring if I an ideal $(r_1 + I) + (r_2 + I) = (r_1 + r_2) + I$ $(r_1 + I)(r_2 + I) = (r_1 r_2) + I$	G/H a group if $H \trianglelefteq G$ $(aH)(bH) = (ab)H$
Isomorphisms and homomorphisms FIT	ϕ: $R \to S$ is a homom if $\phi(r_1 + r_2) = \phi(r_1) + \phi(r_2)$ and $\phi(r_1 r_2) = \phi(r_1)\phi(r_2)$ ϕ is 1-1 $\iff \ker\phi = \{0\}$ $\ker\phi$ is an ideal in R ϕ is an isom if also bijective $R/\ker\phi \cong \text{Im } \phi$	ϕ: $G \to H$ is a homom if $\phi(g_1 g_2) = \phi(g_1)\phi(g_2)$ ϕ is 1-1 $\iff \ker\phi = \{e\}$ $\ker\phi \trianglelefteq G$ ϕ is an isom if also bijective $G/\ker\phi \cong \text{Im } \phi$
Special classes	Commutative rings Cartesian products Rings with unity Integral domains Fields	Abelian groups Cartesian products