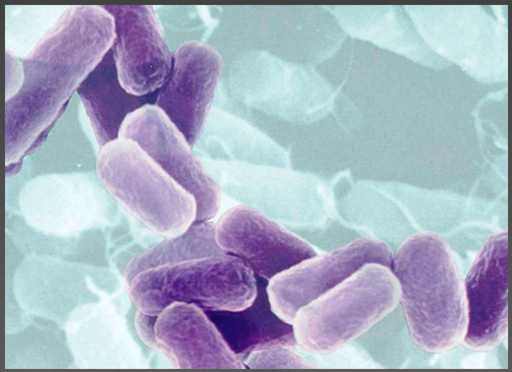
Diffusion of proteins

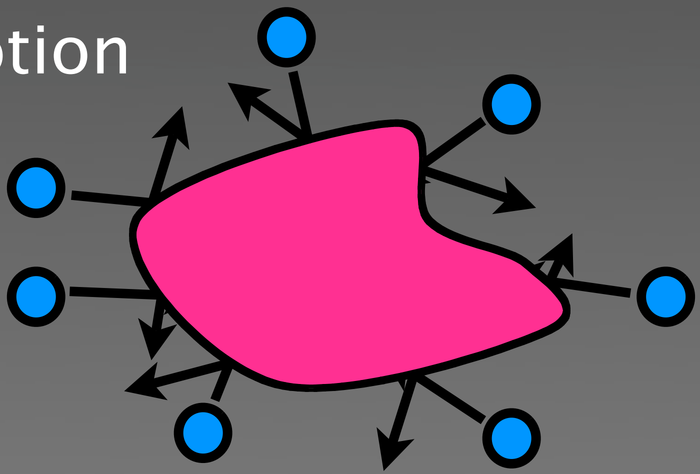
Chemistry 184
Biological Chemistry
Spring 2007

Outline

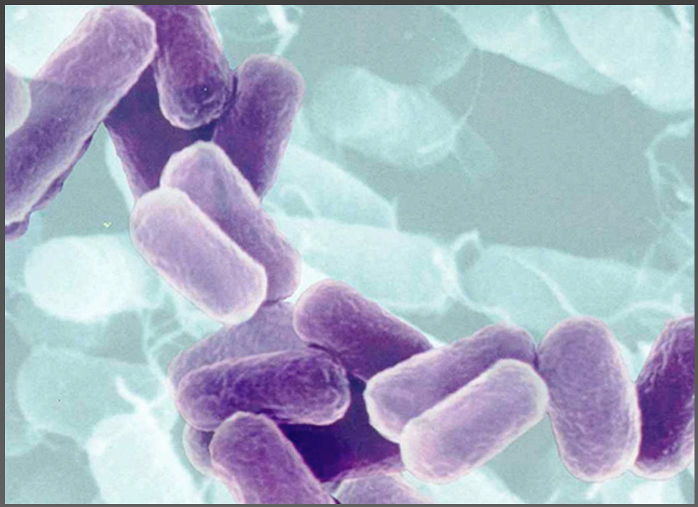
1. Swimming when you are small.



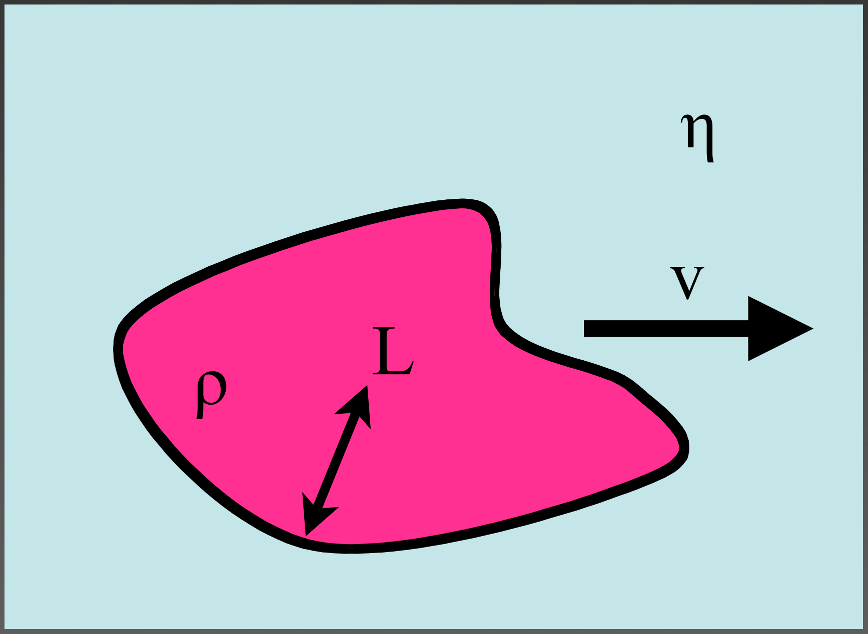
2. Diffusional motion



1. Swimming when you are small



Moving through fluids



ρ - density

η - viscosity

L - length

v - velocity

Two forces affect how you move in a fluid

1) *Inertial forces* - F=ma, keeps things moving

2) *Viscous forces* - F=-bv, drag, makes things stop

Reynolds number describes the ratio of these forces

$$\mathrm{Re} = \frac{F_{inertial}}{F_{viscous}} = \frac{\rho L v}{\eta}$$

O. Reynolds, 1889

Because small objects (small L) also move slowly (small v), Re is drastically smaller for smaller objects.

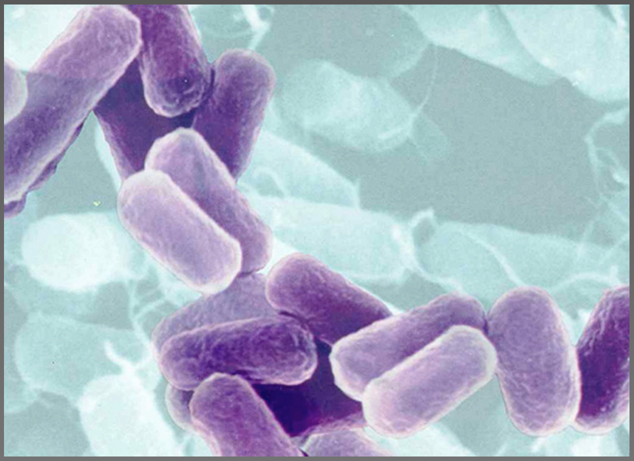
Swimming whale: $\mathrm{Re} = 3 \times 10^{8}$

Swimming bacteria: $\mathrm{Re} = 1 \times 10^{-5}$

Swimming is different at different size scales

$Re = 10^4$

A swimmer can push against water - and then inertia keeps them moving.



$Re = 10^{-5}$

The bacteria cannot use inertia to move.

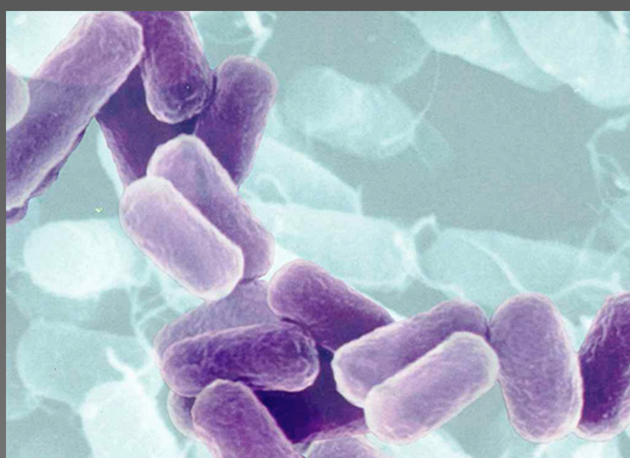
Swimming is different at different size scales

A bacteria can propel itself at v = 2 x 10^{-3} cm/s
What happens when it stops and tries to coast?

$$F = ma = 6\pi\eta r v$$

acceleration of bacteria → ma ; **viscous forces** → $6\pi\eta r v$

$\eta = 10^{-2}$ g/cm sec
$r = 10^{-4}$ cm
$v = 2 \times 10^{-3}$ cm/s
$m = 4 \times 10^{-12}$ g



The bacteria stops moving in 1 μs after traveling 0.01 Å.

It has no history.

You can be low Reynolds number too

$$\mathrm{Re} = \frac{F_{inertial}}{F_{viscous}} = \frac{\rho L v}{\eta}$$

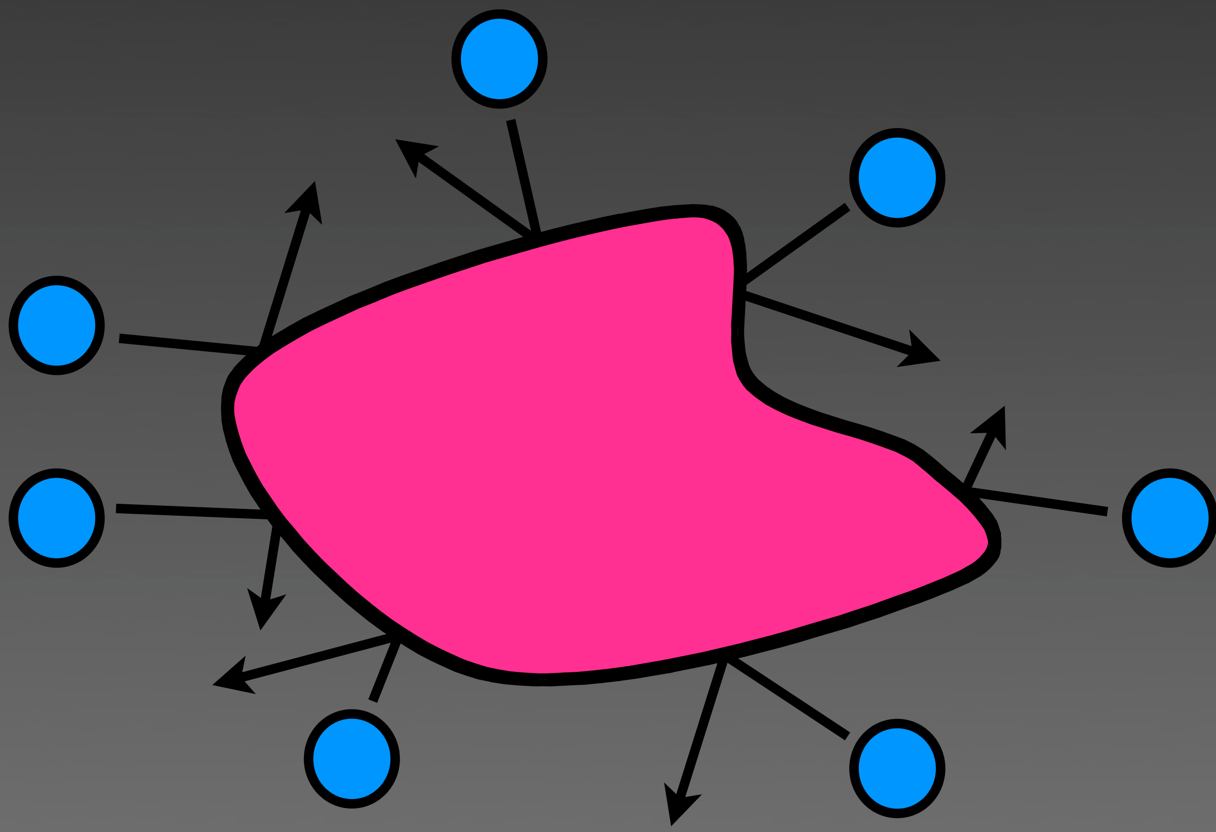
$\eta_{\text{corn syrup}} = 5000\ \eta_{\text{water}}$

~~v = 1 m/s~~ 10^{-6} m/s

$\mathrm{Re}_{you} = 10^9\ \mathrm{Re}_{bacteria}$

If you were to swim the length of Olympic size pool filled with corn syrup over 16 weeks, you would be low Reynolds number.

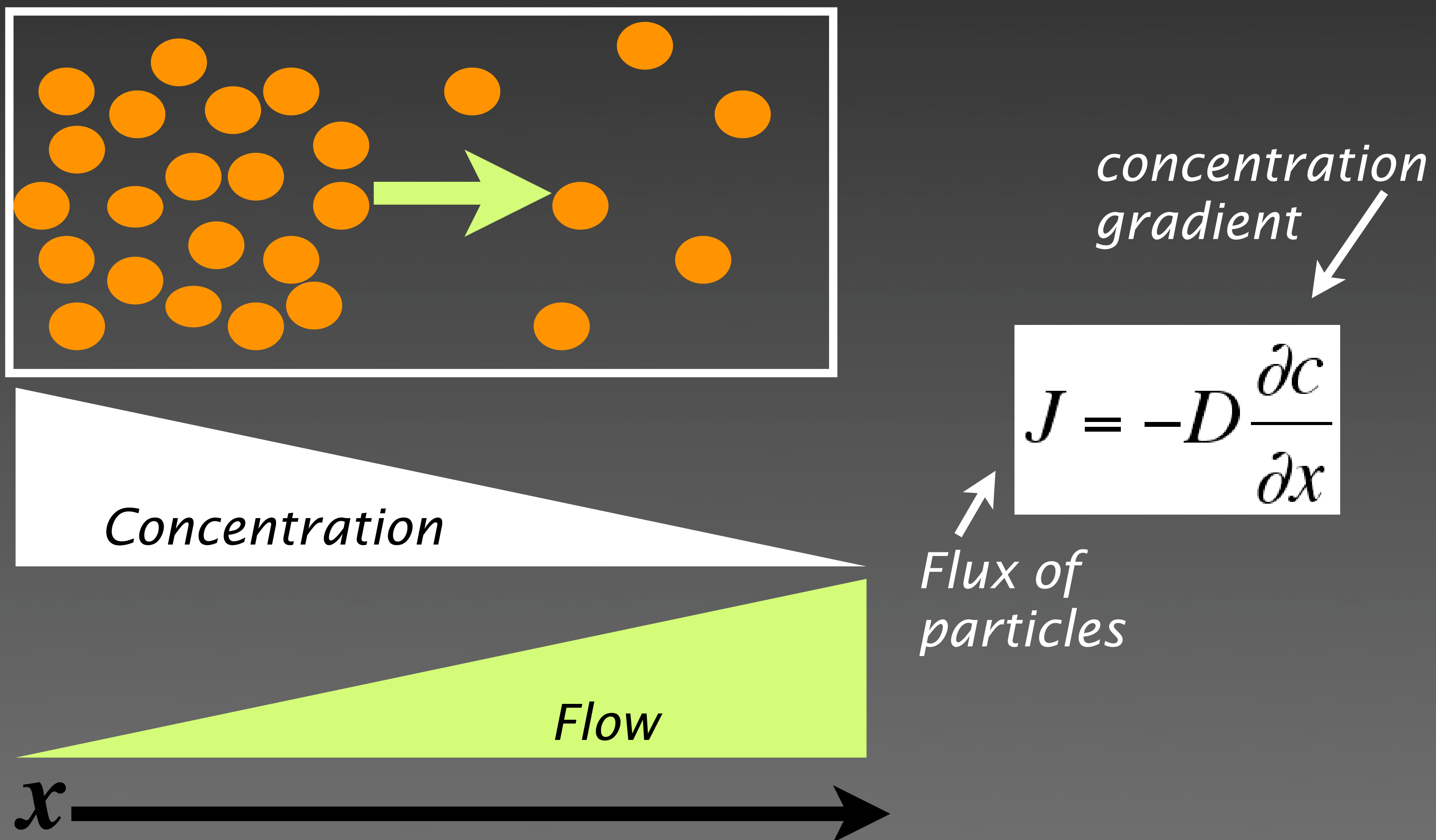
2. Diffusional motion



Brownian motion

Fat droplets in milk (0.5-3 μm)
Dave Walker
www.microscopy-uk.org.uk/dww/home/hombrown.htm

The phenomenological approach



From the diffusion constant, one can predict how concentration gradients change in space and time.

Diffusion from a point source

$$c(x,t) = \frac{c_o}{\sqrt{4\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right)$$

Random Walk

1.

2. Solve for the probability *P* of going a distance *x* after *N* steps

The 1-dimensional random walk

N = # of total steps

$1/2$ = probability of left step

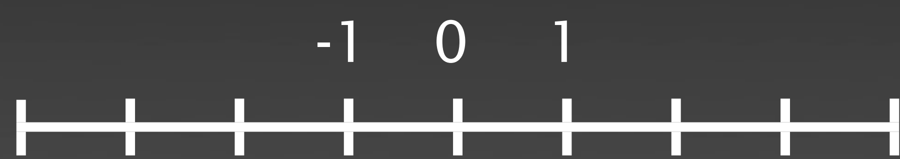
$1/2$ = probability of right step

n_r = # of right steps

n_l = # of left steps

$n_r + n_l = N$

Probabilities and the random walk



Initial					1				
1st step				1/2		1/2			
2nd step			1/4		1/2		1/4		
3rd step		1/8		3/8		3/8		1/8	
4th step	1/16		1/4		3/8		1/4		1/16

Pascal's Triangle

Probability of n_r right steps after N steps total.

$$P(n_r) = \frac{n_r!}{n_r!(N-n_r)!} p^{n_r} q^{N-n_r}$$

$p = q = 1/2$

Probabilities and the random walk

$p = q = 1/2$

$$P(x) = (2\pi N)^{-1/2} \exp\left(\frac{-x^2}{2N}\right)$$

$$\langle x \rangle = 0$$

$$\langle x^2 \rangle = N$$

$$c(x,t) = \frac{n_o}{\sqrt{4\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right)$$

$$N \Leftrightarrow 2Dt$$

$$\langle x^2 \rangle = 2Dt$$

Diffusion from a point source

$$\langle x \rangle = 0$$

$$\langle x^2 \rangle = 2Dt$$

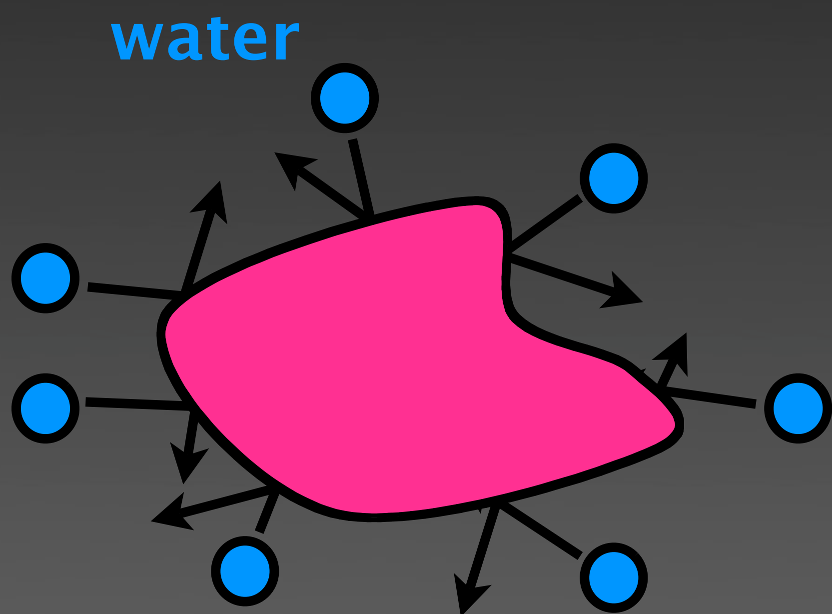
$$\langle r^2 \rangle = 6Dt$$

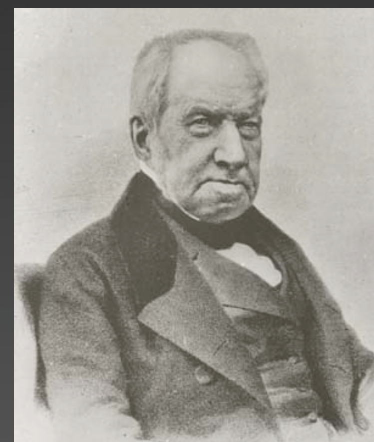
Can we give D meaning?

Assumption - based on kinetic theory

A particle undergoes Brownian motion because it is in a medium that consists of discrete particles undergoing thermal motions.

Microscopic view: Brownian motion





Robert Brown (1827)

Albert Einstein (1905)

Proteins undergo frequent collisions

Thermal forces - forces imparted due to these collisions.

- Randomly directed.
- Results in *thermal motion* - diffusion.
- Because there is no history at low Re, thermal motions at different times are completely independent.