Hydrodynamic Tunneling as a Quantum Analog Experiment

by

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Abstract

General Abstract: As a droplet of silicone oil comes into contact with a vertically oscillating silicone bath, it is propelled upward against gravity into a bouncing motion under the right conditions. At higher accelerations we see a change in motion to a "walking" motion. We are interested in this system because it has similar characteristics to quantum particles. Both systems show a particle of different scale, interacting with their own wave field. We will examine a fluid experiment where we can draw conclusions about how these droplets mimic quantum particle nature. To show these similarities, we will carry out a hydrodynamic tunneling experiment and compare this the quantum case.

Technical Abstract: Quantum mechanics characterizes particles acting in a way unlike objects at the macro scale; quantized particles cannot be described with definite position, but rather by their wave function, giving a probability distribution for the particles location at a certain time. Our hydrodynamic experiment shows similarities to the tunneling system of quantum mechanics. Hydrodynamic tunneling happens when a "walking" droplet approaches a barrier submerged in fluid and overcomes it. However, the droplet can reflect as a consequence of its emitted wave interacting with the barrier.

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1 Introduction

Classical mechanics describes systems as either particles or waves. At the quantum-scale, it is necessary to describe objects as having both particle-like and wave-like properties. Quantum mechanics shows how we can describe a particle using both, known as wave-particle duality. Particles at this scale behave much differently than objects of the macro scale. When looking at the observable measurements (position, momentum etc.) of these particles, they are in all possible quantum states until measured. These states are described by the particle's wave function. A wave function is a complex-valued probability amplitude, from which probabilities of observable measurements can be obtained. These functions are denoted by $\Psi(x,t)$. The probability density is symbolized by

$$P(x)dx = |\Psi^*(x,t)\Psi(x,t)|dx, \tag{1.1}$$

giving a probability density for the particle being between x and $x + dx$.

The idea of wave-particle duality comes from Louis de Broglie, a theoretical physicist at the Faculty of Science at Paris University. In 1924, he proposed his theory that matter behaves like a wave[Com14]. By 1927, two American physicists, Davisson and Germer, confirmed de Broglie's theory of matter waves with the discovery of electron diffraction by crystals[Com14]. de Broglie proposed that, similar to light, which has wave and particle properties, electrons also have wave-like characteristics.

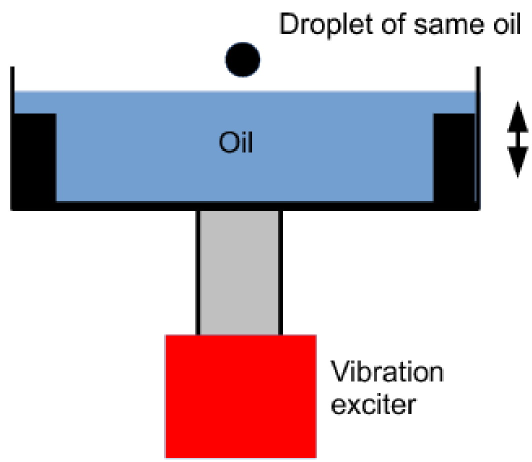

Figure 1.1: Diagram of the setup used for bouncing droplets. The vibration exciter beneath the oil bath is the source of vibration, or oscillation, in the system[BA14].

An interesting phenomenon occurs when a fluid is dropped onto a bath of the same composition. To the naked eye, the droplet appears to coalesce, or merge, with the surface upon contact. As the droplet hits the surface, it either merges immediately, or bounces multiple times, before finally coalescing with the liquid. If the bath is vibrated at just the right frequency, however, the droplet will bounce indefinitely without decreasing in size. Figure 1.1 provides a schematic representation of the system we will be working with. A thin film of air between the drop and the bath propels the drop upward against gravity. Figure 1 shows an actual image for how these droplet will look at the surface of the fluid. The behavior of the droplets within this system is reminiscent to that of quantum particles[Bus15]. The experiment provides a macro scale visual representation of how quantum particles behave.

We will carry out a hydrodynamic tunneling experiment to show the similar nature between quantum particles and these bouncing droplets. The droplets within will be sent toward a submerged barrier, it will either overcome the barrier or be sent back in the opposite direction that it approached. Based on the results we will determine whether this is a good analogous experiment for quantum tunneling. Quantum tunneling will be explained in more depth in the background section of this thesis.

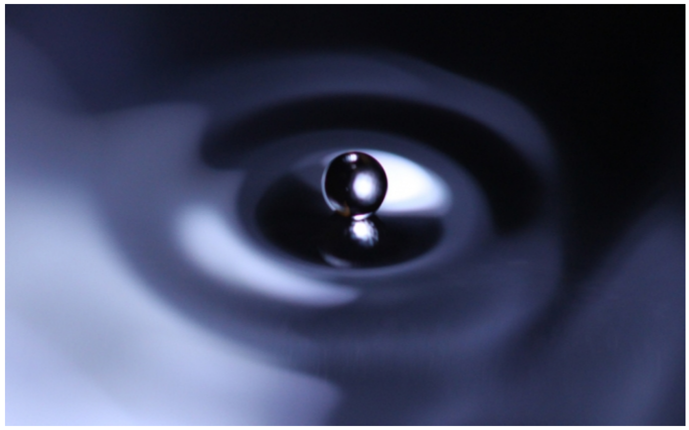

Figure 1.2: A silicone oil droplet suspended above the surface of the fluid. Here we can see a wave being emitted from the drop[Har13]

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Chapter 2 describes the parameters and apparatus used, and will compare tunneling in our model with quantum tunneling. Chapter 3 describes our experimental setup, including the apparatus and software used. Chapter 4 will show the data that we collected from the hydrodynamic tunneling trials. In Chapter 5 we will analyze the results and determine how well our experimental results compare to the theoretical results from quantum mechanical tunneling.

2 Background

Chapter 2 will cover two main topics, bouncing droplet dynamics and tunneling. This will give us the necessary information to understand how different droplet motion will be utilized to carry out the experiment.

2.1 Droplet Dynamics

In this experiment, we use a bath of oil of density ρ, kinematic viscosity v, surface tension σ, depth H. The bath is vertically driven at amplitude A_0 and frequency $f = \frac{\omega}{2\pi}$. The effective gravity in the bath's reference frame is $g + \gamma \sin \omega t$, where g is the gravitational acceleration and the driving acceleration is $\gamma = A_0\omega^2$ [Bus15].

2.1.1 Faraday Waves

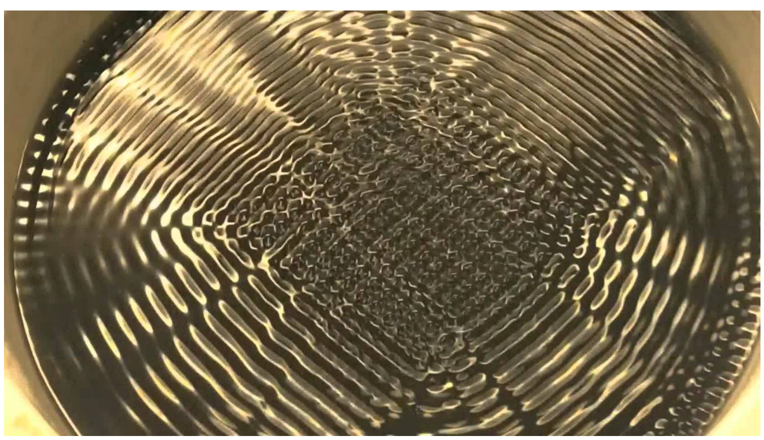

Figure 2.1: Faraday waves occur at frequencies above the Faraday threshold. These are unstable waves that oscillate at half of the driving frequency. From [youtube].

The fluid in the vibrating bath is driven by a shaker from beneath. In the absence of an oil drop at a certain frequency and acceleration, the fluid will produce standing waves, known as Faraday waves [GRS14]. The fluid appears distorted and forms small ripples at the surface, as shown in Figure 2.1. The behavior of the fluid, going between Faraday and quiescent waves, depends on the γ term, whether it be high or low. When γ increases to a critical amplitude, Faraday waves are produced; the transition point is called the Faraday threshold γ_F. Two forms of motion by the droplets are observed at γ below the Faraday threshold; these are bouncing and walking movement.

2.1.2 Bouncing Droplets

As mentioned earlier, at low γ, droplets in the experiment will coalesce and merge with the fluid bath. However, at sufficient γ bouncing motion occurs. A droplet in this environment will interact with the bath and be propelled upward and bounce indefinitely. These bouncing droplets stay in the same lateral position, only changing vertically in position. After each bounce, the droplet emits a localized damped wave. This means, the wave that comes from the drop each bounce dissipates and by the next impact, the bath effectively has a flat surface for the drop to bounce again. The dampening of the droplets wave is what allows the drop to remain in a fixed lateral position. In considering the dynamics of these drops, they behave like a linear spring, with spring constant proportional to σ [Bus15].

2.1.3 Walking Droplets

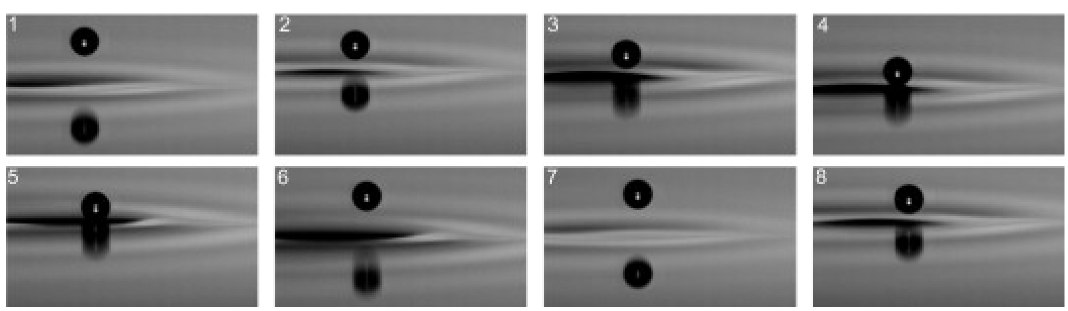

Figure 2.2: A walking droplet interacting with its own wave field, being propelled laterally across the oil bath [CLSM14].

Walking droplets are a specific type of bouncing droplet which occur at γ between the bouncing and Faraday thresholds. After impact on the surface of the bath, the emitted wave is not as damped as the bouncing droplets because of the increase in acceleration. Now, the droplet interacts with the emitted wave field from earlier bounces. At each bounce, the droplet does not hit a flat surface, instead it hits the wave at a non-centered point. This causes the droplet to move laterally, being propelled by its own wave field as shown in Figure 2.2.

2.2 Tunneling

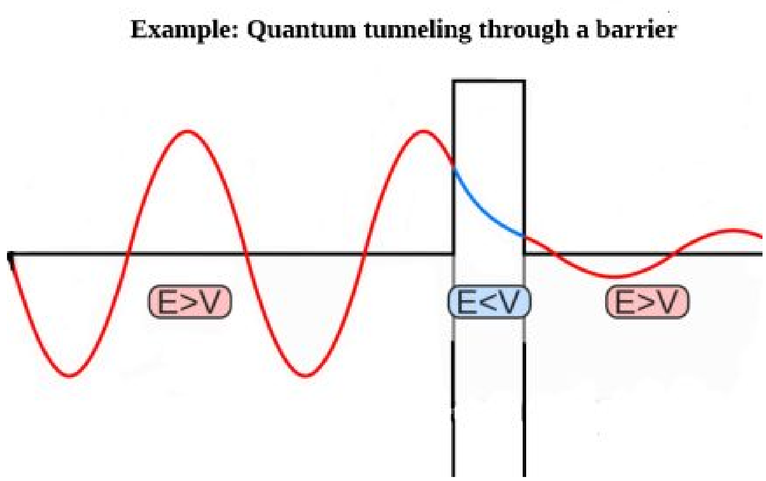

Figure 2.3: Here, we have a particle that is close to a barrier. We can see that when it is close, the most probable location of the particle is before the wall. When you look past the barrier to the right, there is a small part of the wave function. This shows the small probability that the particle tunnels through the barrier [Her15].

2.2.1 Quantum Theory

Quantum tunneling in short, is the idea that a particle has a small probability of crossing through a barrier. Classically, this is impossible; if an object does not have enough energy to cross a barrier, then it will not do so [Her15]. In quantum mechanics particles can be described by their wave function. As mentioned above, a particle's wave function gives a probability distribution for its location. When a particle approaches a barrier (one with appropriate width to move through), although the most probable locations are seen in front of the barrier, at that point, there is a small probability that the particle is behind the barrier (see Figure 2.3). However, there are limitations. If the barrier is too thick, the particle will not be able to pass through. The probability of the particle passing through the barrier is dependent on the size and height of the barrier.

2.2.2 Hydrodynamic Case

Due to the scale of our experiment, it is not possible to try and get droplet through the barrier. Instead, we will have a submerged barrier just beneath the path of the drop. As the droplet approaches the barrier, there are reflected waves that can guide it in the opposite direction.

2.2.3 Uncertainty Principle

First thought of by German physicist Werner Heisenberg, the uncertainty principle became partially understood. He found that, when observing a particle's position and velocity, you can shine a light on it and view the reflection. This method works only for the macroscopic scale, not the atomic scale. This is because, when the light hits the particle, it is moved by the photons of light. The reflected light allows us to determine the particle's position, but the velocity of the particle becomes unknown because of this movement. So now we see that the act of observing a particle changes the state of that particle.

3 Experimental Method

In chapter 3, we will discuss the design of our experiment. Walking droplets have very specific system parameters that need to be met to view their motion. We use multiple detectors to allow us to get these exact values of driving acceleration and frequency, for example.

3.1 Apparatus

Figure 3.1 is our experimental setup. We see an oil droplet generator suspended above the oil bath. Beneath the bath is a vibration generator which oscillates, causing the excited motion of the drops. Fixed to the generator is an accelerometer, which keeps track of the acceleration that the bath exhibits.

3.1.1 Oil bath

The oil bath is made from aluminum and was milled into a dish shape. Figure 3.1 shows the dish above the vibration generator. The depth to the bottom of the dish is 13 mm. There are small screw holes on the bottom of the dish, where we connected the accelerometer housing piece. We will be using silicone oil of viscosity 20 cSt(centistokes). The barrier is glued down to the bottom of the dish, submerged by a small layer of oil.

3.1.2 Barrier

The barrier is made of HIPS (high impact polystyrene) filament printed from a LulzBot Taz 6 3D printer. The structure in Figure 3.2 shows the sketch for our barrier. The disk shape is used to fit tightly within the oil tray. There is a diamond cutout in the center and a square cutout at the top of the disk [EFMC09]. The shape of the diamond is key while running our tests; this allows us to constrain

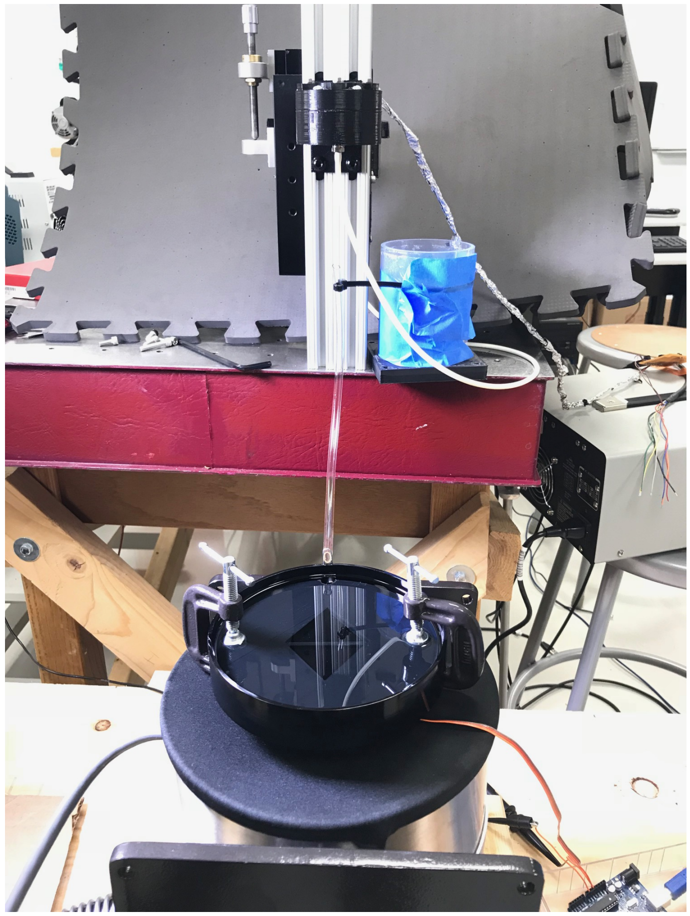

Figure 3.1: Full setup of the system. The droplet generator is fixed above the oil tray. When dispensed, the droplet is guided by a piece of rubber tubing onto the oil tray to reduce coalescence.

the droplet when it is in the oil tray. When the droplet is in this region, its emitted waves interact with the corners of the diamond which propel the drop in a trajectory perpendicular to the thin bar in the center. This thin bar is the actual object that we send our droplet toward to tunnel over.

Three barriers were printed with varying heights 2.75 mm, 3.00 mm, and 3.25 mm. The oil 9 mm deep, from the surface of the oil to the bottom of the dish. The disk and diamond cutout connected to the barrier bar are 6 mm thick, they lie 3 mm below the surface. The depth of the fluid above the barrier varies with the change in barrier height.

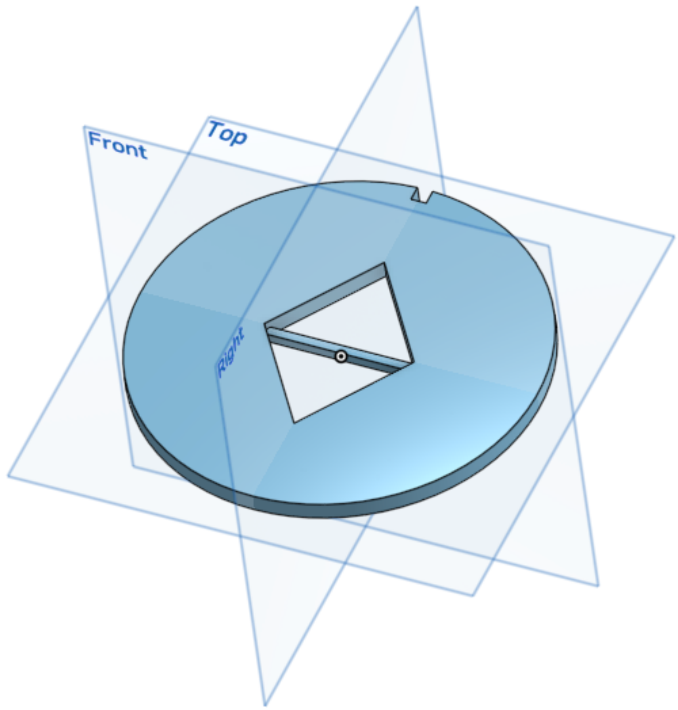

Figure 3.2: This figure shows the barrier that designed using an online CAD program called Onshape. The barriers varied in the height of the bar seen in the middle of the diamond shaped cutout. The depth of the cutout does not change with the variance in height of the bar.

3.1.3 Oil Droplet Generator

The oil droplet generator is extremely important in our setup. To gather repeatable data, we must have this generator to produce droplets of the same size each time we run the experiment. The dynamical behavior of the droplets depends on this consistency. Figure 3.3 shows the model that we based our own droplet generator on, we did not use the pump because it turned out to be unnecessary in our design. Silicone oil is filled into both the fluid reservoir and fluid chamber. The fluid chamber hangs above the reservoir at Δh, the difference in height causes hydrostatic pressure at the nozzle. The translation stage allows us to adjust the height of the reservoir in small increments. If the height difference is too large, the oil will retract back into the tubing towards the reservoir and cause air bubbles to form in the line. We must not have these bubbles because they will compress against the liquid, instead of a solely liquid ejection. The opposite effect occurs

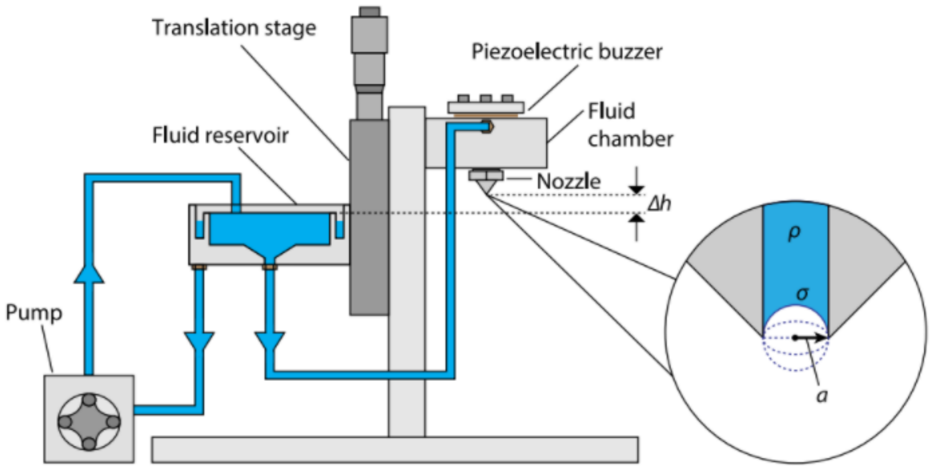

Figure 3.3: The oil droplet generator adapted from Harris, Daniel M., Tanya Liu, and John WM Bush. "A low-cost, precise piezoelectric droplet-on-demand generator." Experiments in Fluids 56.4 (2015): 83.

when Δh is small, the fluid will flow out of the nozzle. Static equilibrium of the fluid about the nozzle happens when,

$$|\Delta h| \leq 2\sigma/\rho g a. \tag{3.1}$$

Here,σ is the surface tension, ρ is the fluid density, and a is the radius of the nozzle.

Attached to the top of the fluid chamber is a piezoelectric buzzer. The buzzer is run with 40 V from an Instek power supply. Connected to this setup is a circuit which consists of an Arduino mini micro-controller, trigger button, an H-bridge circuit. The micro-controller allows us to connect the buzzer and trigger together, giving us an effective droplet producer at the push of a button. Before the button is pushed, the buzzer remains at a constant negative voltage, then when pressed, is quickly provided with a square voltage wave form of positive voltage. The rapid application of positive voltage causes the buzzer to contract, pushing out a droplet, then return to negative contraction which brings more fluid back into the chamber. The H-bridge circuit permits us to make the sudden change in voltage possible within 500-700 μs. We produced drops of 1 mm, with 10% variability. This large variability comes from the mode of measurement we used. An object that we already know the size of was placed next to our drops and visually measured.

3.1.4 Accelerometer

To measure the acceleration of the oil bath, we used an Adafruit ADXL326 triple-axis accelerometer. This was directly fixed to the bottom of the oil bath. Figure 3.4 shows this with a cutout hole to view the detector. The accelerometer is connected to a separate micro-controller, capable of giving us readings of acceleration in all three axes. For our purposes, we only wired the z-axis pin.

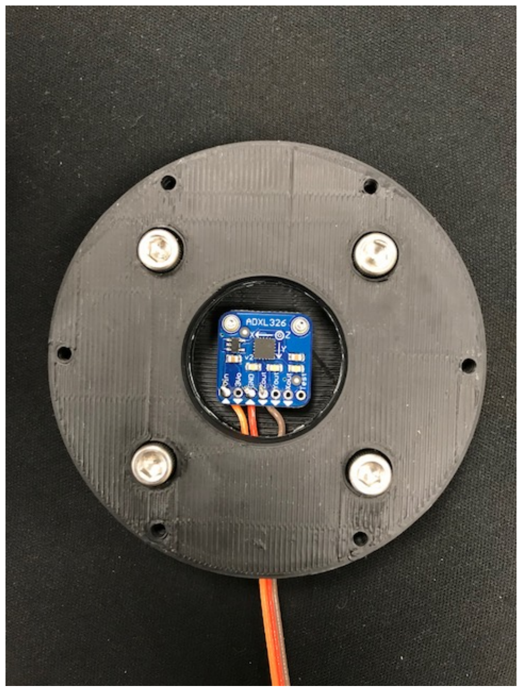

Figure 3.4: The ADXL326 triple-axis accelerometer, secured to the top of the vibration generator. Readings of acceleration are given in the z direction.

3.1.5 Camera

We used a high speed camera to ensure that we were getting droplets that were consistent in size. To do so, we connected our camera to a circuit board and programed the micro-controller to capture images of droplet leaving the nozzle. Once the camera was in sync with capturing when the droplet reached 4 cm below the nozzle, we were able to compare multiple images to see that they were repeatable in size.

3.1.6 Vibration Generator

We use a VG-80A vibration generator to excite the oil dish. It is capable of outputting 80 lbsf (pounds force). Due to the lengthy run time of our experiment, the generator is equipped with a cooling fan to prevent any overheating. Additional heat worth noting would affect the viscosity and surface tension of our fluid, which would throw off our results.

Function Generator/Oscilloscope

Our function generator is a Tektronix CFG253. The range it can reach is 0.03Hz to 3MHz. The generator can produce square, triangle, and sine waves. For our purposes, we will only be using sine waves. To view the measurements that the accelerometer detected, we paired it with a Hantek DSO5202P 200MHz oscilloscope. The oscilloscope is capable of making a large range of measurements, we specifically looked at the frequency (Hz) and the peak to peak acceleration (mV). To gather our acceleration as a multiple of units of gravity ($9.8m/s^2$), we took our values in mV and used the relation,

$$C \cdot \frac{\gamma}{g} = \frac{A_{scope}}{2}, \tag{3.2}$$

where C is the unitless sensitivity value 51 for the accelerometer, and A_{scope} is the peak to peak acceleration. To isolate the $\frac{\gamma}{g}$ term, we divide the peak acceleration by the sensitivity.

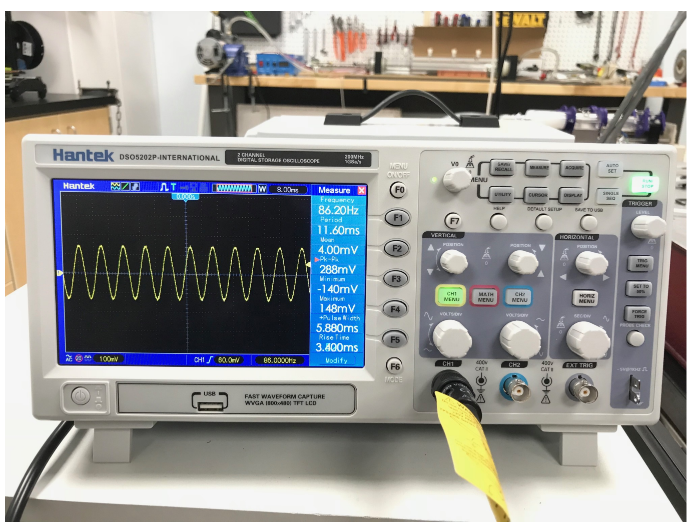

Figure 3.5: The Hantek oscilloscope. Readings appear on the right side of the screen in the light blue region. The device also has a display of the sinusoidal oscillation produced by the function generator.

3.2 Procedure

After constructing the entire setup with sensors in place, we were able to begin trying to produce repeatable bouncing and walking droplets. Our approach is modeled after the findings in John WM. Bush "Pilot-wave hydrodynamics." [Bus15]. First, we familiarized ourselves with how droplets behave in the bath by powering the vibration generator, getting it to oscillate below the Faraday wave threshold, and flicked the surface of the oil with a wire. This method made droplets of varying size, not important for our case, but we were able to see how these randomly sized droplets interacted with not only the bath, but with each other. Adjusting the frequency and amplitude at random, we saw droplets bouncing, walking, coalescing, and even pairs of drops orbiting around a center point.

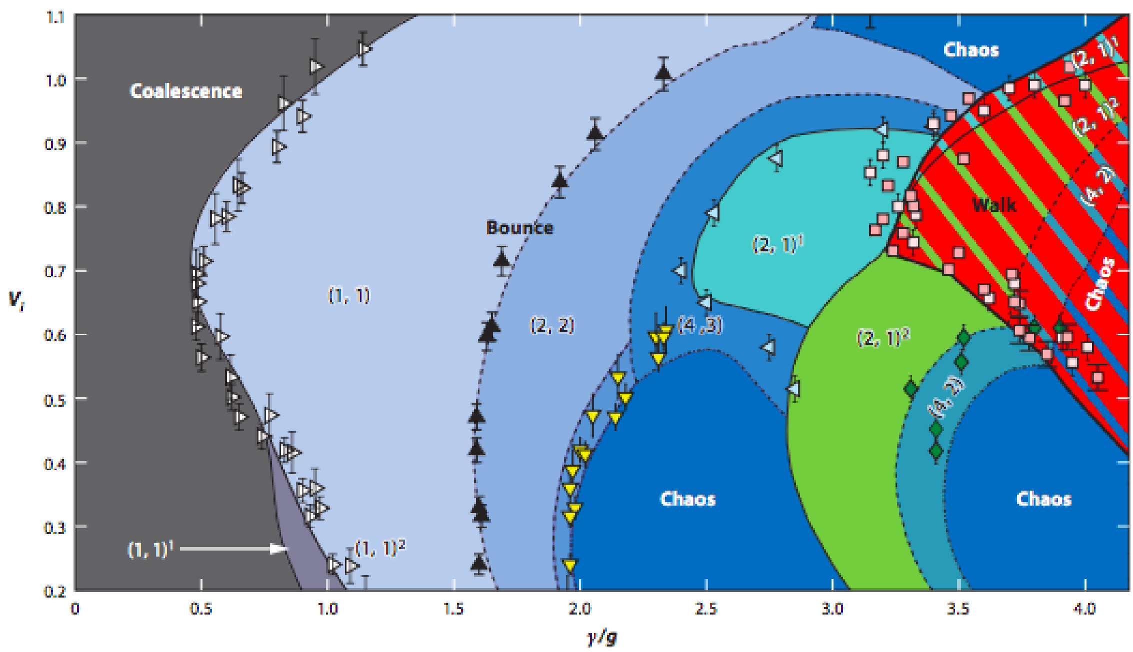

Figure 3.6: A regime diagram for the droplet motion adapted from [Bus15]. The different numbers shown on the graph in parentheses, are representations of different bouncing states. An explanation for these is given for these states in our procedure. The y axis is the vibration number and the x axis is the value of our γ in terms of gravity.

In Figure 3.6, we see a graph of droplet motion on a vibration number (V_i) vs driving acceleration ($\frac{\gamma}{g}$). The vibration number is the ratio of the forcing frequency and the drop's natural oscillation frequency, as it compresses and decompresses from the impact from the bath given by

$$V_i = \omega\sqrt{\rho a^3/\sigma}, \quad (3.3)$$

where a is the radius of the droplet. In the same image, we see these pairs of numbers at different locations of motion denoted (m, n).

These are the varying bouncing states, m is the forcing periods, n is the number of bounces. Referring back to our model, the driving frequency of the bath is set at 80 Hz. We were not able to produce a suitable environment for the droplets to walk with driving acceleration above 2.94 γ/g. This is because, at the modeled 80 Hz and driving acceleration of 3.2 γ/g, droplets would coalesce. To account for this we tried increasing the amplitude, but due to the sensitivity of the function generator, the bath would undergo Faraday waves. We carried out the experiment using a range of frequencies between 86-94 Hz. The increase in frequency allowed for enough driving acceleration without transitioning to Faraday waves. Our walking regime parameters differed from those in Figure 3.6. Our driving acceleration range for walkers was 2.86-2.94 γ/g, with an increase in vibration number scale due to the increase in frequency.

4 Results

This chapter will examine how walking droplets interact with a barrier submerged in oil. The droplets will undergo two forms of activity with the barrier, reflection and "tunneling" over the barrier. Each trial for tunneling will be dependent on the oscillation of the bath and barrier size, which will determine the outcome of the drop. The probability of droplet tunneling decreases exponentially with the barrier width, but will increase as γ reaches the Faraday threshold [Bus15].

As mentioned earlier, the droplets will approach barriers of heights 2.75 mm, 3.00 mm, and 3.25mm. After the dynamical parameters of the oil bath were set, we took videos of walking droplets for each trial. With the 2.75 mm barrier, the droplet crosses over with little disturbance in its trajectory. In the 3.00 mm case, we begin to see notable interaction with the barrier. Instead of crossing with ease, the droplet bounces in front of the barrier for a moment, then traces along the barrier, and finally bounces over then continues walking motion. Lastly, with the 3.25 mm block, we saw the most interaction and time at the barrier. As shown in Figure 4.1 the drop seemed to be stuck bouncing directly above the barrier for a significant period.

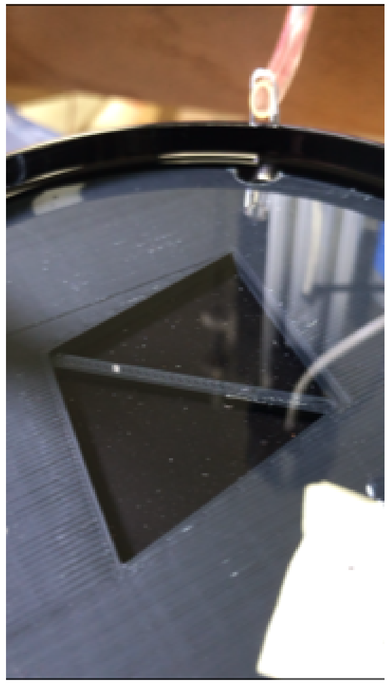

Figure 4.1: An image from the video of a droplet interacting with the 3.25 mm barrier. Within the video, the drop seems to be in a trapped state as it tries to overcome the bar. This barrier took the longest to see tunneling affects.

5 Conclusion

We were able to successfully have droplets within our experiment tunnel over the fixed barrier. We were not able to to get enough conclusive data to say that the two tunneling cases emulated each other significantly. Based on that results that we found, we were able draw a correlation between height of the barrier and the interaction time that the droplet has with the barrier. The two quantities mentioned are directly proportional to each other. There is a point at which the height is too large for a drop to tunnel over. We would see an incoming droplet just bounce off of the barrier being pushed in the opposite direction due to the reflection force.

5.1 Future Work

There were limitations within our experiment. The precision in measuring the droplet size could have been done better. The current method is to compare images of multiple droplets to check that they are consistent; then we use an object of a known size and compare the droplet to the size of the object. Instead, we would like to use the images taken at the nozzle and run a python code that is able to detect the parameter of the drop. Then, we will have code that can give us a pixel value for the detected region and give us the droplet size with variability of 1%, rather than 10%. The tunneling experiment is only one of multiple quantum investigations that we can run to show how our hydrodynamic setup can emulate quantum particles. Specifically, we would be very interested in the single slit particle diffraction experiment. The aim would be to get a probability distribution for our droplets location through a slit, which would look similar to the diffraction grating we see with light on a screen.

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