Effects of Perturbation Amplitude on the Subcritical Transition to Turbulence in Taylor-Couette Flows

by

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Abstract

General Abstract: Turbulence in fluids is not well understood but has many implications for the scientific and industrial communities. By utilizing a Taylor-Couette system, a fluid dynamics system that allows for controlled inducing of turbulence, we studied how turbulence evolves from disturbances to smooth flows. The Taylor-Couette system consists of two independently-rotating concentric cylinders with a fluid filled gap. By rotating the outer cylinder at varying rotational velocities, the Reynolds number, a value that describes a flow's behavior, was adjusted. At set Reynolds numbers, the smooth Taylor-Couette flow was agitated by an injection system which produces a jet of fluid from the inner cylinder of the system. By perturbing the laminar flow, it is possible to draw conclusions regarding how strong a perturbation is required to induce turbulence at specific Reynolds numbers. By conducting this experiment, conclusions can then be drawn regarding how natural-world fluid systems of varying Reynolds numbers react to perturbations.

Technical Abstract: Turbulence is characterized by complex spatiotemporal dynamics within a fluid and because of this extreme complexity, turbulence is not a well understood phenomenon. This complexity is caused by flows constantly changing velocity and direction making for chaos. Alternatively, laminar flows have fluid that smoothly slides past itself. The Reynolds number is a dimensionless quantity that tells whether a fluid's motion is viscosity or inertia dominated. As the Reynolds number increases, the spatiotemporal dynamics of a flow become increasingly complex and become more likely to give way to turbulence supercritically. A subcritical transition to turbulence is when a stable laminar flow suddenly erupts into turbulence due to a perturbation or disturbance in a flow. By utilizing a Taylor-Couette system, a system composed of two concentric and independently-rotating cylinders with a fluid filled gap, this question can be analyzed. The Taylor-Couette produces flows by rotating one or both of the cylinders. By rotating only the outer cylinder, a linearly stable flow is created. By perturbing this flow, subcritical transitions to turbulence were analyzed to understand the relationship between the minimum perturbation amplitude to induce turbulence and the Reynolds numbers at which that turbulence was induced.

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1 Introduction

In the natural world, the state of fluids is largely dominated by turbulence, a chaotic state with complex spatiotemporal dynamics. In many cases, this turbulence erupts suddenly from a previously smooth, or laminar, flow. The transition to turbulence in a majority of these flows is not well understood, but in experimental settings with controlled environments, it can be studied. One such controlled environment is the Taylor-Couette system which was utilized in the studies documented in this thesis.

One of the greatest questions in the area of fluid dynamics is "why and how does turbulence evolve from seemingly stable laminar flows?" There is very little understanding with regards to the transition to turbulence, but different types of transitions are identifiable. The supercritical transition occurs through bifurcations leading to ever-increasing levels of complexity in the flows until the flow is so complex that it is turbulent. A subcritical transition to turbulence, however, occurs nearly instantaneously in settings that are not particularly conducive to the evolution of turbulence and is thus fairly mysterious.

During a supercritical transition to turbulence, a value called the Reynolds number Re increases along with the complexity of the flow whereas the complexity of a subcritical transition stagnates until it seemingly randomly erupts into turbulence via a subcritical transition. The Reynolds number is a key parameter when studying flows as it describes whether a flow is dominated by the viscosity of the fluid or the inertia of the fluid's movement. To briefly describe how the Reynolds number affects a flow, it can be said that as the Reynolds number for a flow increases, the complexity of the flow increases. An easy to visualize example of this is a sink faucet. If the faucet is not fully open, the water will drip out slowly. As the faucet is opened more, the water will begin to flow out in a clear and smooth line. Usually, when the faucet is opened all the way, the water will appear bubbly, less smooth, and chaotic; this is turbulence.

Aside from purely scientific reasons, understanding the transition to turbulence has vast practical applications. Due to the complexity of a turbulent fluid's dynamics, it is very good at mixing. Therefore, it would be helpful in settings that require mass homogenization such as ensuring that a chemical reaction has taken place fully or that a mixture or solution has reached maximum homogeneity. Turbulence is also a player in determining the amount of drag on objects that move through fluids. Because drag forces oppose the movement of an object in the direction that it is travelling, it reduces efficiency in vehicles such as cars, planes, trains, and boats. In industrial settings, being able to induce or halt the evolution of turbulence is an invaluable resource.

To assist in the understanding of turbulence, developments in the apparatus of studying fluid dynamics have been made. One such apparatus is the Taylor-Couette system. Comprised of two rotating concentric cylinders with a fluid-filled space between them, the Taylor-Couette system is an excellent resource for studying transitions to turbulence. The Taylor-Couette system is capable of creating a variety of flow types and producing turbulence both supercritically and subcritically [Col65]. Because of these capabilities, the Taylor-Couette system is very versatile in research applications. The variety of conditions that can be created in a Taylor-Couette system are primarily results of the multitudinous parameters that can be adjusted on the system.

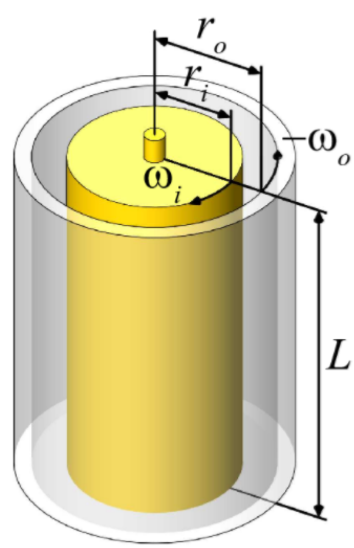

Figure 1.1: The Taylor-Couette system's physical attributes are numerous making for a versatile system that is able to exhibit multiple behaviors dependent upon its configuration. The parameters of the Taylor-Couette system are the inner and outer radii (r_i and r_0), the length of the cylinders (L),the axial aspect ratio ($L/2r_{i/o}$) and the angular velocities of the inner and outer cylinders (ω_i and ω_o). A final parameter is given by d which describes the gap width between the cylinders ($r_o - r_i$) [MJG+14].

Figure 1.1 shows a schematic of a Taylor-Couette system displaying the inner and outer cylinders along with the dimensions of the system. All of the parameters associated with the Taylor-Couette system (other than L) affect the Reynolds number which is a key player in the study of transitions to turbulence. A hole on the inner cylinder is present in the iteration of the Taylor-Couette system that will be in use throughout this experiment. From the hole, jets of fluid can be injected into the system to disturb the flow. Henceforth known as perturbations, these disturbances of variable strength are introduced into otherwise laminar flows and are used to induce turbulence through a subcritical transition. By triggering turbulence in this way, we can see how the amplitude of the smallest perturbation needed to induce turbulence scales with the Reynolds number which helps to distinguish between different proposed mechanisms for transition.

Another feature of the Taylor-Couette system used for this experiment is the addition of ring-shaped end caps attached to the inside of the outer cylinder to adjust the height dimension of the system. The end caps will be on either side of the jet to create the area that will be under examination in this experiment. By using the jet in conjunction with the end caps, turbulence can be brought about in a relatively controlled manner, and this is what the experiment outlined in this thesis will focus on.

The rest of this thesis is structured as follows: Chapter 2 will discuss the Reynolds number in depth and give an insight into how the Taylor-Couette system functions. It will also discuss laminar and turbulent flows, how they relate to the Reynolds number, and how they are created in a Taylor-Couette system. Chapter 3 will explain the experimental apparatus of the experiment and the experimental procedure in detail. Chapter 4 will discuss the results of this experiment and what future work may be made to produce more and better data. Lastly, Chapter 5 concludes the thesis with results and a brief discussion of the experiment's progress.

2 Background

In this chapter, the Taylor-Couette system will be explained in more detail. Laminar flows and turbulence will also be discussed along with how they are related to and affected by the Reynolds number. Examples of previous work will be included to better illustrate this material.

2.1 The Navier-Stokes Equation and Reynolds Number

The Reynolds number is a dimensionless value used to describe whether a fluid's movement is dominated by viscosity or inertia. This value can be derived from the Navier-Stokes equation in the context of an incompressible fluid. The Navier-Stokes equation is essentially Newton's Second Law $F = ma$ or $F/V = a\rho$ in the context of fluid dynamics and is given in dimensional form by

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right) = -\nabla p + \mu \nabla^2 u \tag{2.1}$$

where ρ is the density of the fluid, V is the volume of the fluid, and $\mathbf{u}$ is the velocity of the fluid. This yields the pressure gradient and the viscous stresses respectively which equate to the force over unit volume. Essentially, this equation describes the forces on a small volume of fluid within a flow.

To non-dimensionalize this equation, non-dimensional scaling factors can be chosen for each of the parameters. So, in Equation 2.1, because $\mathbf{u}$ can be thought of as a function of x and t, scaling factors must also be chosen for these variables. The scaling factors usually take the form of

$$\mathbf{u} = \mathbf{u}'U,\ x = x'L,\ t = t'\frac{L}{U},\ \text{and}\, p = p'P \tag{2.2}$$

where p is the local pressure that the volume of fluid is experiencing from the flow around it. The volume may experience stresses normal to its surface or viscous stresses caused by the flows around it pulling or pushing it parallel to its surface. Once the scaling factors are plugged in for their respective values, the constants of the equation can be eliminated yielding the form

$$\frac{\partial \mathbf{u'}}{\partial t} + (\mathbf{u'} \cdot \nabla')\mathbf{u'} = -\frac{P}{\rho U^2}\nabla' p' + \frac{\mu}{\rho U L}\nabla^2 \mathbf{u'} \tag{2.3}$$

where μ is the dynamic viscosity of the fluid.

By allowing P to equal ρU^2 and letting $Re = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$ where ν is the kinematic viscosity of the fluid, the final non-dimensional form of the Navier-Stokes equation with the Reynolds number included is

$$\frac{\partial \mathbf{u'}}{\partial t} + (\mathbf{u}' \cdot \nabla')\mathbf{u}' = -\nabla p' + \frac{1}{Re}\nabla^2 \mathbf{u}' \tag{2.4}$$

Despite the Reynolds number being an effective quantifiable trait of a fluid mechanical system and a fluid's flow, whether or not it defines a critical point at which a fluid becomes turbulent is contextual to the type of flow. Reynolds numbers are used to predict when or if a fluid will become turbulent, but there are cases in which a high Reynolds number does not correspond to turbulence. For example, in studies of a viscous fluid flowing through circular pipes, the range of Reynolds numbers at which the pipe flow transitioned to turbulence has a very large span. Reynolds numbers for the transition to turbulence in pipe flows have been found to range from near 1000 to greater than 3000 [Eck07]. This means that for some Reynolds numbers that should have indicated the presence of turbulence, a laminar flow was present and vice versa, which is very problematic if one wishes to define a critical Reynolds number for the transition to turbulence in a system.

In the context of a Taylor-Couette system, the Reynolds numbers are

$$Re_i = \frac{r_i \omega_i d}{\nu}, Re_o = \frac{r_o \omega_o d}{\nu} \tag{2.5}$$

where Re_i and Re_o represent the inner and outer cylinders' Reynolds numbers, respectively. The inner and outer radii are represented by r_i and r_o, the angular velocities of the cylinders by ω_i and ω_o, the width of the gap between the cylinders d, and the kinematic viscosity of the fluid by ν.

In a flow with viscosity-dominated movement, indicating a Reynolds number lower than one, the fluid is likely to be laminar. This could be due to a high viscosity, or in the context of a Taylor-Couette system, a low angular velocity or small gap between the cylinders. However, in the case of a high Reynolds number, the nature of the fluid's motion is more suitable for the evolution of turbulence.

A laminar flow state occurs when a fluid is able to cleanly and smoothly slide past itself in layers [Bat67]. These layers can be conceptualized by imagining a very viscous fluid such as molasses. If molasses was sliding down an incline, layers of the fluid would be moving over other layers slowly and smoothly. The flow of the molasses would be very predictable and not exciting to witness, and this is due to its dynamics' low level of spatiotemporal complexity. By and large, a laminar flow will remain laminar unless the flow conditions are altered. In theory, some laminar flows will remain laminar for any Reynolds number.

In a fluid exhibiting complex turbulent dynamics, the Reynolds number is relatively high, certainly greater than one, indicating an inertia dominated flow. The dynamics of a turbulent fluid are complex because different regions of the fluid are continually changing in both velocity and direction in a chaotic manner.

2.2 Transitions to Turbulence

While not precisely the objective of the experiments outlined in this thesis, the discussion of a critical Reynolds number offers a helpful perspective in understanding the transition to turbulence. As the Reynolds number is increased, the fluid's state becomes gradually more dominated by the inertia of the fluid rather than being kept laminar by its viscosity. Eventually, when the Reynolds number becomes high enough and reaches a critical point, the fluid will transition from a laminar flow to turbulence [ME05]. A supercritical transition to turbulence is characterized by the presence of increasingly complex spatiotemporal dynamics in the fluid as the Reynolds number increases and approaches a value that corresponds to the fluid being entirely characterized as turbulent. Supercritical transitions are observed in a multitude of flows such as specific Taylor-Couette flows and in certain instances of plane channel flows [STS91] [ALS86].

Unlike the transition to turbulence observed in supercritical scenarios, subcritical transitions occur without a gradual increase in the complexity of the fluid's dynamics. Instead, the fluid remains stably laminar with no evolution of regions of complex spatiotemporal dynamics and then, when perturbed at a Reynolds number that allows turbulence to form, erupts into turbulence. As discussed earlier, however, the Reynolds number can vary greatly. This makes it extremely difficult both to predict turbulence in such flows and understand why and how turbulence evolves from a laminar flow in such an abrupt fashion. Subcritical transitions occur in flows such as pipe flows and Taylor-Couette flows dominated by rotation of the outer cylinder when perturbations are introduced to the flows [BC12].

]Laminar versus Turbulent Flows

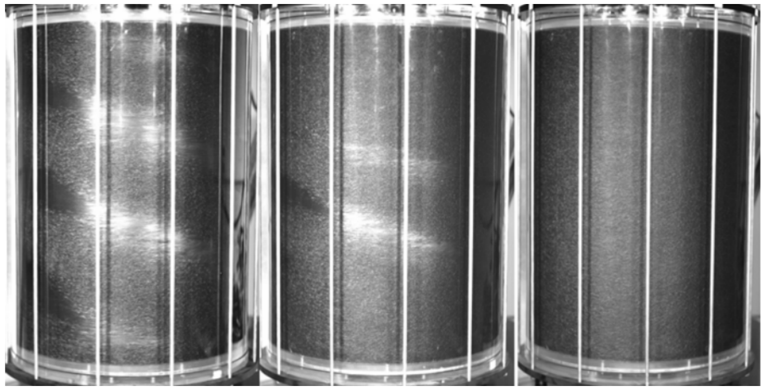

Figure 2.1: The subcritical transition to turbulence evolves from a perturbation and, in a Taylor-Couette system can form patterns such as spiral turbulence. Spiral turbulence can occur from perturbations in large Taylor-Couette systems when the outer cylinder rotates independently [BC12].

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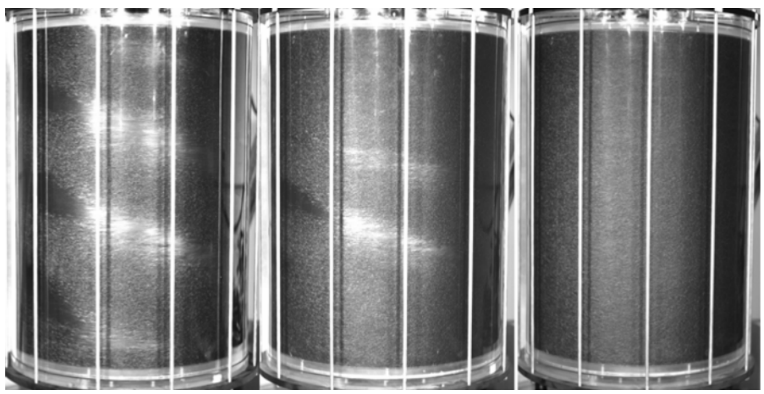

Figure 2.2: The subcritical transition to turbulence evolves from a perturbation and, in a Taylor-Couette system can form patterns such as spiral turbulence. Spiral turbulence can occur from perturbations in large Taylor-Couette systems when the outer cylinder rotates independently [BC12].

When perturbed, a Taylor-Couette flow can experience a subcritical transition to turbulence. In Figure 2.2, a subcritical transition to turbulence appears in the form of spiral turbulence. In the case of this experiment, a perturbation was induced into a Taylor-Couette flow created by rotating the outer cylinder. Spiral turbulence was created and then dissipated in the photo on the right [BC12].

2.4 Taylor-Couette Flows

Taylor-Couette flows are defined by any flow occurring between two concentric rotating cylinders [Tay23]. They are unlike most other types of flows due to their ability to undergo both supercritical and subcritical transitions to turbulence.

An example of the Taylor-Couette system's ability to transition to turbulence supercritically is the Taylor-Couette flow involving Taylor vortices.

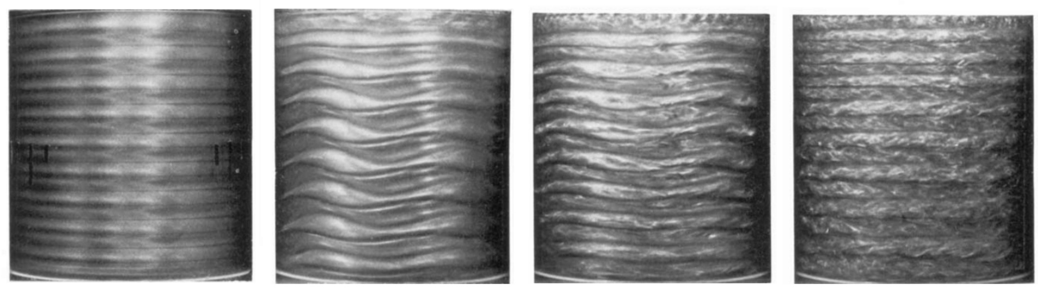

Figure 2.3: Taylor vortices appear in horizontal bands along the axis of rotation in a Taylor-Couette flow when the inner cylinder rotates, and the outer cylinder remains stationary. This creates an outward centrifugal force on the fluid causing it to get pushed to the outer wall and rotate back in on itself. In the case of this figure, the Reynolds number is increasing from left to right creating increasingly complex flows. The increase in the Reynolds number is achieved by increasing the angular velocity of the inner cylinder [FSG79].

Illustrated in Figure 2.3, when the inner cylinder of a Taylor-Couette system is rotated, it centrifugally forces the fluid outward into the wall of the outer cylinder. Because the fluid has no other place to go, it spirals in on itself to form stratified bands. When the Reynolds number Re_i for the inner cylinder is increased by increasing the angular velocity of the inner cylinder, the bands become increasingly complex and time dependent. This means that over different points in time, the same spatial point within the fluid will now have a different flow behavior. Eventually, any recognizable pattern is lost, and the fluid behaves with completely turbulent dynamics.

On the other side of the spectrum, a Taylor-Couette flow can quickly become turbulent from a laminar flow when the outer cylinder is rotated by itself [Col65]. While this type of Taylor-Couette flow does not exhibit any bands or other interesting dynamic formations, it provides a blank canvas of laminarity which one can utilize for the purposes of studying transitions to turbulence.

In the case of this experiment, we will perturb a Taylor-Couette flow produced by the rotation of just the outer cylinder with a jet of fluid from the inner cylinder to induce turbulence that will then subcritically evolve into a greater region of turbulence. We will consider the minimum amplitude perturbation to induce turbulence at a given Reynolds number to understand the stability of the flow and when turbulence can be expected for certain Reynolds numbers. These experiments follow up on those conducted by Dr. Daniel Borrero-Echeverry for his 2014 Ph.D. thesis dissertation [BE14].

3 Methods

This chapter will go into depth about the setup of the experimental apparatus and performing the experiment. Specifically, it will discuss the perturbation system, the Taylor-Couette system's physical attributes, the flow visualization methods, and the stepper motor automation. This chapter will also discuss the experimental procedure.

3.1 Apparatus

The apparatus used to conduct this experiment has many variable parameters and has been used in experiments by scientists such as G.I. Taylor to analyze the stability of flows with solid boundaries [Tay23]. The particular Taylor-Couette system used for this experiment allows the inner and outer cylinders to rotate independently of one another and the perturbation strength to be adjusted.

3.1.1 Taylor-Couette System

The Taylor-Couette system is comprised of two independently rotating concentric cylinders. The inner cylinder is a milled piece of aluminum that is anodized to create a smooth surface with a radius of 4.72 cm. It was anodized black to improve visualization of flows against it. Within the inner cylinder is a tubing to a port on its outer perimeter from which fluid can be injected to create perturbations in the gap between the two cylinders. The outer cylinder is made of clear polished glass allowing for visualization of the flows within it. It has an inner radius of 5.94 cm. The bottom of the inner cylinder is on a bearing that is fitted into the bottom end cap, and around the bottom perimeter of the outer cylinder is an end cap that is mounted on a ring bearing. These two separate bearings allow for independent rotation of each cylinder. At both the base and top of the outer cylinder are o-rings to create a waterproof seal and prevent leakage. A valve is located at the bottom of the system to allow for filling the system with the desired fluid.

Figure 3.1: Within the outer cylinder, the inner cylinder can be seen though clouded by the shaving cream and water mixture. On top of the Taylor-Couette system's bracings are the stepper motors and timing pulley system. The tubing that can be seen running from the top of the system is connected to the perturbation system.

Attached to the inside of the outer cylinder are rings that allow for the adjustment of the size of the area that will be under analysis. To isolate the area between them, the difference between the rings' inner and outer radii is only slightly smaller than the gap width between the inner and outer cylinders of the Taylor-Couette system. The rings have vertical holes drilled in them so that fluid may pass through them when the system is filled with them in place. Otherwise, bubbles would form, and the system would not fill as efficiently and uniformly.

3.1.2 Stepper Motors

The cylinders were independently driven by their own Phidgets 86STH156 NEMA-34 Bipolar Gearless Large Stepper motor. The two motors were controlled by a Phidgets Control Panel Program. This program was necessary to communicate with the PhidgetStepper Bipolar HC board and allowed for easy adjustment of the motors' independent velocities and accelerations. The stepper motors had a resolution of 200 steps per revolution, meaning that the motor will proceed through 200 increments of 1.8° for one full rotation of its axle. The Phidgets program works in increments of microsteps, however, meaning that the step number is increased by a factor of sixteen. Therefore, there are effectively 3200 steps per revolution of each motor's axle and therefore 0.1125° per step.

Each motor was connected to its respective cylinder via a drive train consisting of a timing pulley on the motor, a timing belt, and a timing pulley on top of the cylinder. The timing pulleys on the cylinders are larger than those on the motors' axles and create a gear ratio of 60:18 or effectively 10:3. For every ten turns of the motor, the respective cylinder will rotate three times. This was necessary to provide a greater torque to the cylinders as they must move the fluid along with their own mass. This also gives more precise control of the cylinder's velocities because large changes in motor velocity equated to smaller changes in cylinder velocity.

3.1.3 Perturbation System

As mentioned earlier, the inner cylinder has a port that allows for the introduction of perturbations to the Taylor-Couette flow. To create said perturbations, there must be sufficient pressure passing through the port, or else the perturbations would not affect the flow in a substantial or meaningful way. To create enough pressure and be able to make precise adjustments to it, we used a Harvard Apparatus Syringe Infusion 22 syringe pump, shown in Figure 3.2 pump to induce the perturbations. The syringe pump holds two syringes with 3D-printed mounts and is controlled via a Python program for automation purposes. From the syringes, Tygon vinyl tubing runs to a solenoid valve that leads to either a waste reservoir or the top of the inner cylinder where it meets a pressure transducer and then a rotary joint. The pressure transducer reads the strength of the perturbation, and the rotary joint allows the inner cylinder to rotate while the top of the joint remained stationary.

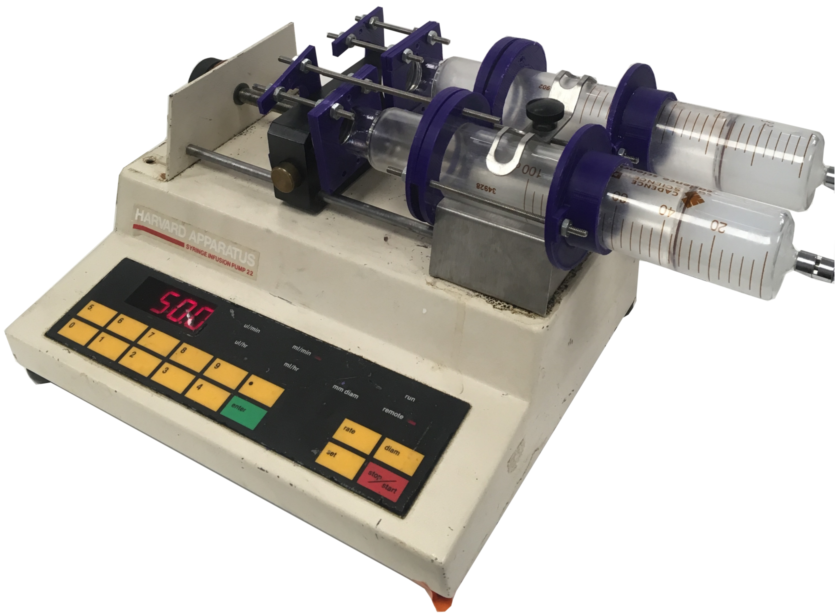

Figure 3.2: The syringe pump can hold two syringes and is driven by a block on a threaded rod that rotates with the stepper motor. On the front of the pump, a number pad is present for manual input of the diameter of the syringes and the desired flow rate. The units of the flow rate are also adjustable giving the option to set it in microliters or milliliters per hour or per minute. The purple parts on the syringes are 3-D printed mounts to hold the syringes in place during both pumping and withdrawing fluid from the system.

Initially, the syringe pump's timing pulley did not provide a fast enough flow rate to produce vigorous perturbations that could actually cause turbulence. To remedy this, I 3-D printed a new timing pulley to change the gear ratio from 2:1 to 1:1. This worked with smaller syringes, but to increase the flow rate further, larger syringes were implemented. Unfortunately, the 1:1 gear ratio did not provide enough torque to the pump, and the motor would consistently stall with the larger syringes in place, so the original gear ratio was returned to. This return was successful in that it provided enough torque to the pump to push the syringe's plungers, and the larger syringes with an outer diameter of 38.9 mm allowed for a greater flow rate to produce strong enough perturbations to disturb the Taylor-Couette flow.

To make the perturbations consistent and reproducible, a three-way solenoid valve was used. The fluid pumped from the syringes goes into a waste reservoir to allow the syringe pump to get up to speed and to allow for a discrete perturbation to be introduced when the valve is activated. Upon activation of the valve, the valve to the reservoir closes, and the valve to the injection system of the inner cylinder is opened, thus directing the fluid to the port where a perturbation then occurs in the gap between the two cylinders. The valve was activated by a solenoid driver transistor circuit that was powered by a 9 V power DC power supply, and the relay was able to be triggered remotely by an Arduino Uno that was controlled by Python code.

3.2 Computer Programs

To operate the Taylor-Couette system as a whole, some degree of programming or software was necessary. Each component has a separate program. Ideally, these would all be condensed into a single program for full automation.

3.2.1 Coding and Programming

The syringe pump was controlled through a Python program that utilized a serial to USB connection. The pump itself requires specific language to operate, and I used Python to create a program that would communicate with it via an RS-232 connection. This Python program contains commands to set the diameter of the syringes, the flow rate of the syringes, a function to initiate pumping, a function to run the pump in reverse to draw fluid back into the syringes, and a function to halt the pump's motor. The way the program was written allows for automation, which is necessary to collect enough data to draw meaningful conclusions.

The transistor-basdd solenoid driver circuit for the valve was controlled via an Arduino UNO R3 board. The board by itself could not be automated to the full extent required, so it was necessary to access the Arduino with a Python code. The Python code contains a function that sends a voltage to the relay for a duration controllable with millisecond precision. When the relay has a small voltage applied to it, the valve opens, and a perturbation occurs. After the voltage is no longer supplied, the valve closes and directs the fluid back towards the waste reservoir. This program and the program used to control the syringe pump can be coordinated into one program that allows for the two functions to be timed to repeatedly produce uniform perturbations.

The stepper motors are controlled by the Phidget22 Control Panel. This program detected the Phidgets board and the motors connected to it. Within the program, a GUI allows for easy control of the motor's speeds, accelerations, and allows for the motors to be engaged.

3.2.2 Flow Visualization

To visualize the flow, we used a mixture of shaving cream and deionized water. Deionized water was chosen because it has impurities removed and therefore provides a consistent and accurate viscosity. Specifically Barbasol brand shaving cream was used due to its lack of extra features for a shaving experience and the fact it contains stearic acid crystals which are reflective to light and are very close to being neutrally buoyant in water. The stearic acid crystals allow for a rheoscopic fluid to be created when the shaving cream is added to water. A rheoscopic fluid has particles that align themselves when in a particular flow that can be seen when light reflects off them. The buoyancy is helpful because rather than sinking or floating, the stearic acid crystal particles stay suspended in the water. This allows the system to sit for periods of time while maintaining its visual integrity. If the particles were to sink or float, the mixture would need to be disturbed to re-disperse the particles evenly in the water. The mixture of shaving cream and water is shown in Figure 3.3, and it visualizes Taylor vortices that were created in the Taylor-Couette system used in this experiment.

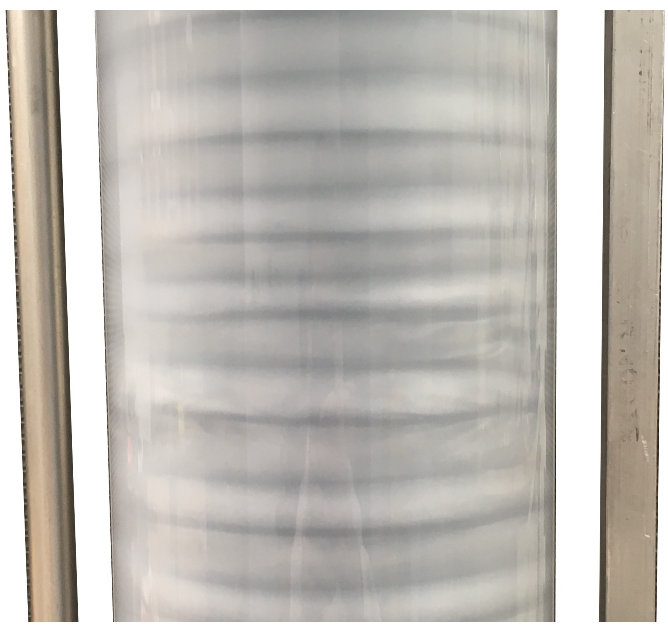

Figure 3.3: The Taylor-Couette system's Taylor vortices are easily visible when the rheoscopic shaving cream and water mixture is used to visualize the flows. Just as other rheoscopic fluids have shown, this Taylor-Couette flow produces bands in its lower Reynolds number and less spatiotemporally complex states. This shows that the shaving cream and water mixture is a viable method for flow visualization.

This mixture was made by adding the shaving cream to water in a near half and half volumetric ratio. After stirring vigorously, the mixture is left to sit at which point a foam begins to form on the surface of the fluid beneath it. This foam actually consists of stearic acid particles that were unable to mix into the water. After letting the fluid sit for roughly an hour, the fluid below the foam is siphoned into a separate container. This method of collecting the fluid was chosen because siphoning the fluid did not disturb the foam on top and obtained the fluid below that already had the maximum concentration of shaving cream possible without it separating into foam. At this point, an equal volume of water was poured onto the foam, and the entire process was repeated until the fluid no longer formed a foam.

3.3 Experimental Methods

Manual adjustments to the perturbation and Taylor-Coutte systems allowed for rudimentary data to be taken. Ideally, the experiment would have been fully automated, but time constraints and limitations regarding Python's ability to process multiple functions at once did not allow for this. Furthermore, source code for controlling the stepper motors was not conducive to editing for the needs of this experiment.

3.3.1 Perturbation Reproducibility

Before beginning the experiment, the perturbation system needed to be refined so that consistent perturbation strengths could be achieved. To assure that the perturbation amplitude was consistent between each perturbation, a pressure transducer was used to measure the pressure induced by each injection of fluid into the Taylor-Couette system.

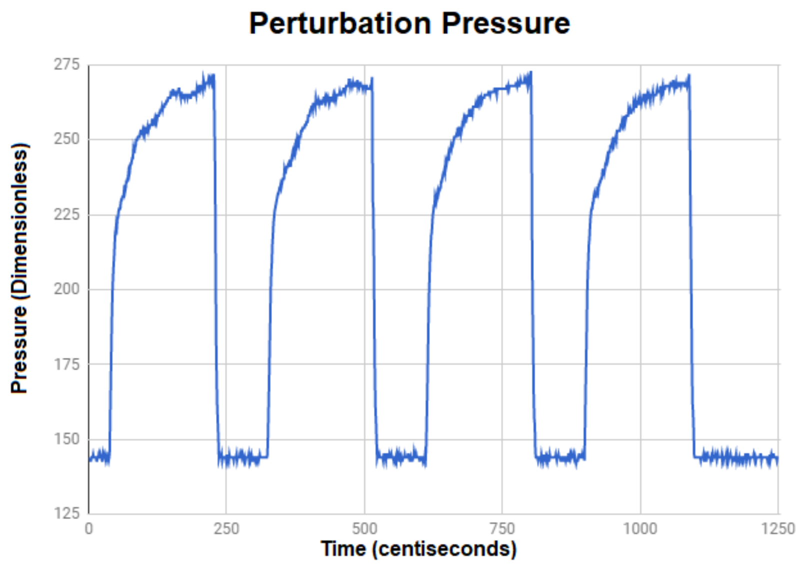

Figure 3.4: In this graph, the pressure of four test perturbations against time is illustrated. The pressure is shown in a dimensionless number that was output by the Arduino connected to the pressure transducer. A clear saw tooth shape is visible in each perturbation's pressure, and each perturbation was measured at the same flow rate.

In Figure 3.4, the shape of the function that describes the perturbation increases sharply and then in a curve until reaching the maximum amplitude at which point the pressure drops off suddenly. The drop in pressure is due to the perturbation system's valve being closed. Because the perturbation's pressure graph showed an increase that was neither linear nor offered a distinctly sustained maximum amplitude prior to the valve being closed, the perturbation strength was difficult to quantify. To further complicate measuring the perturbation amplitude with pressure, when a perturbation was sustained, the amplitude fluctuated in an oscillatory fashion. Ideally, the amplitude would rise sharply, remain flat, and then fall quickly. Because of the ambiguities in the perturbations' pressures, it was decided to use the flow rate of the syringe pump (measured in milliliters per minute) to quantify the perturbation amplitude.

Despite being difficult to measure accurately, it can be seen in Figure 3.4 that the perturbations' maximum amplitudes remained consistent for a given flow rate. This was promising because it meant that the perturbations were still somewhat replicable and that each fluid injection would be similar enough so as to produce nearly identical perturbations. This replicability was necessary because perturbations of differing amplitudes will induce turbulence differently.

3.3.2 Setup of Taylor-Couette System

After it was confirmed that perturbations of reproducible amplitude could be produced, the Taylor-Couette system could be set up to be ready for perturbations. This process included setting the boundary rings within the outer cylinder. This was done before the system was fully assembled. The rings were attached to the outer cylinder with set screws that pressed into the inside of the outer cylinder.

After the entire system was assembled, the gap between the cylinders was filled with the shaving cream and water mixture by siphoning it through the valve on the bottom of the system. By filling from the bottom up, fewer bubbles were created. To further prevent bubbles, the inner cylinder was rotated at a low velocity to disturb the fluid as it rose. Bubbles were not desired because they could induce turbulence thus creating an environment in which it would be unclear whether the perturbations were inducing turbulence.

The perturbation system also needed to be cleared of bubbles because bubbles would alter the perturbation amplitude if in the tubing leading from the syringe pump to the Taylor-Couette system. This was done by withdrawing fluid from the gap between the cylinders into the two syringes. This sucked out any bubbles that may have been in the line. After this, the valves to the syringes were closed, and the air in the syringes was pushed out so that no bubbles could be injected into the tubing.

3.3.3 Conducting the Experiment

Once the Taylor-Couette system and perturbation system were bled of all air bubbles, the experiment was ready to be conducted. To conduct the experiment, I first decided on an angular velocity for the outer cylinder. This was done through the Phidgets stepper motor software. The angular velocity was set to 35,000 microsteps per second. This equates to 3.316 rotations per second. The inner cylinder's velocity was set to zero so that its stepper motor would be engaged in place but motionless. This was done as a precautionary measure to prevent the inner cylinder from rotating due to forces from the fluid's movement as this could create undesired flow types.

The outer cylinder's velocity was increased incrementally so that the stepper motor would have enough torque to drive the cylinder. After the outer cylinder was up to speed, the flow rate of the syringe pump was set to 50 ml/min. At this point, a digital metronome was employed at a ticking rate of 120 beats per minute. The metronome was used to time the perturbations' durations because the automated relay to trigger the Python program that controlled the valve that would induce a perturbation stopped functioning correctly.

Once the outer cylinder was rotating and the metronome was keeping time, the syringe pump was initiated. The fluid was pumped into the reservoir until the relay was triggered by inserting a wire into an Arduino board which supplied the relay with 5 V. The triggering of the relay opened the valve and produced a perturbation in the Taylor-Couette flow. The wire was inserted into the Arduino board for three beats of the metronome which translates to perturbations with a 1.5 second duration. This duration was chosen because it was long enough to produce the full perturbation amplitude while not sustaining that amplitude for very long. Essentially, it was the minimum duration to achieve the maximum amplitude for a given perturbation because as seen in Figure 3.4, the perturbation pressure increases on a curved function. If the perturbation's duration was too long, turbulence became more likely.

Upon opening the valve and introducing the perturbation to the Taylor-Couette flow, the flow was observed for the presence of turbulence. With a flow rate of 50 ml/min, turbulence occurred for every trial, so the perturbation amplitude was reduced. Figure 3.5 illustrates the difference between the laminar Taylor-Couette flow and turbulence induced by a perturbation. The rheoscopic fluid created by the shaving cream in the water provided visualizations that were undeniably turbulent regions in the Taylor-Couette flow.

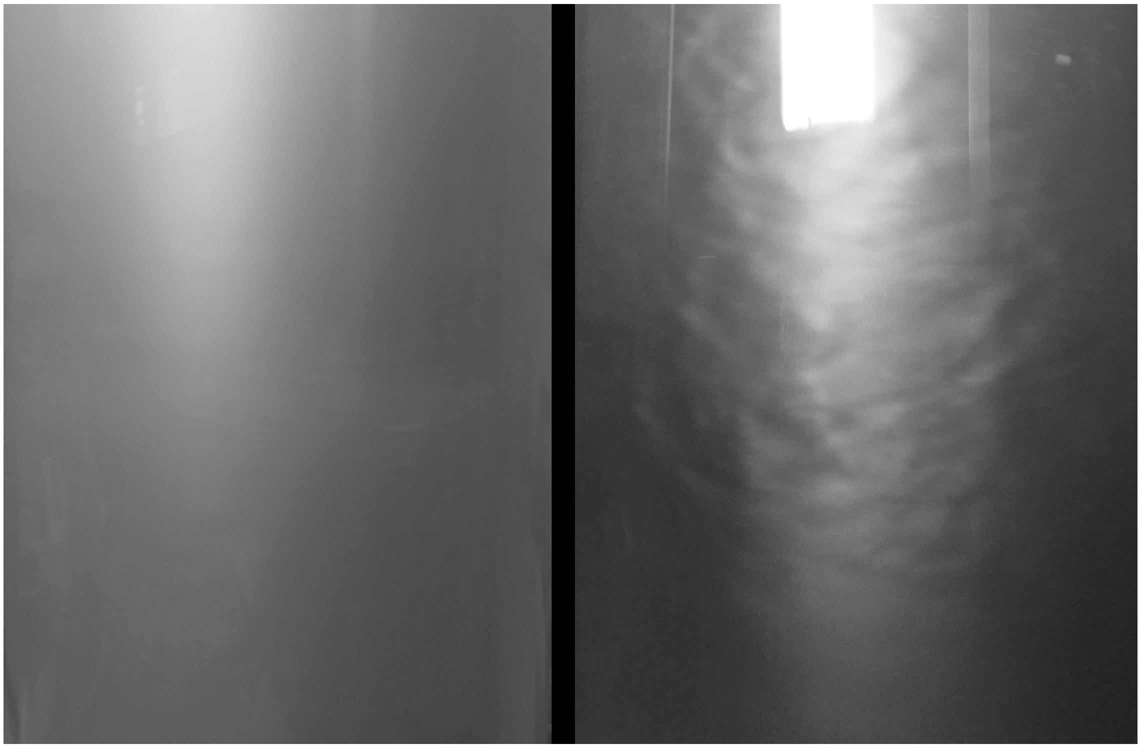

Figure 3.5: The subcritical transition to turbulence is achieved via a perturbation. On the left side of this figure is the laminar Taylor-Couette flow and on the right is turbulence that was caused by a perturbation. This turbulence lasted for roughly fifteen seconds. It is observable that the right image has more complex spatiotemporal dynamics because of the uneven shading. Note: the large white block near the top of the image is a reflection and does not describe any of the dynamics of the flow.

In increments of 2.5 ml/min, the perturbation amplitude was tested and then reduced until the perturbation no longer produced turbulence in the Taylor-Couette flow. At the point at which the perturbation amplitude did not produce turbulence, the next highest amplitude was taken as the minimum perturbation amplitude to induce turbulence in Taylor-Couette flows. Ideally, this process would have been completed for many different angular velocities, and therefore different Reynolds numbers, as this would have provided more data to draw conclusions from. From this trial, it was found that the minimum perturbation amplitude required to induce turbulence at $Re_0 = 15091.0842$ was 37.5 ml/min. This Reynolds number comes from the radius of the outer cylinder being 59.4 mm, the gap width d being 12.2 mm, and the rotational velocity being 3.316 rotations per second.

4 Results and Discussion

This chapter will discuss the results of conducting this experiment and where future researchers may want to focus their efforts to achieve more conclusive data.

4.1 Results

Though conducting this experiment did not provide data that could be used to analyze the effects of perturbation amplitude on the subcritical transition to turbulence in Taylor-Couette flows, it is still worth discussing what amalgamated through conducting this research and what the data that may be collected in the future will mean.

The one point of data that was collected shows that at a Reynolds number of $Re_0 = 15091.0842$, the minimum perturbation amplitude necessary for a subcritical transition to take place is 37.5 ml/min. 37.5 ml/min is essentially a dribble of fluid from the syringes, so this perturbation amplitude was quite small. Because the Reynolds number was so high, however, this is all it took to induce a subcritical transition to turbulence. It can be speculated that for larger Reynolds numbers, it will take smaller perturbation amplitudes to induce turbulence and for small Reynolds numbers, larger perturbation amplitudes will be required to induce turbulence. This is because as the Reynolds number increases, the environment becomes more conducive to the evolution of turbulence despite an otherwise laminar flow.

Another result of this experiment was uncovering issues with the perturbation system. Illustrated in Figure 3.4, the function by which the perturbation amplitude rises is not measurable by means of a single quantity. Rather, it would require defining a function which is not a viable or convenient means of measuring the perturbation amplitude. Refining the perturbation system is a project that future researchers will need to continue work on if they wish to collect data in enough quality and quantity to draw meaningful conclusions regarding the subcritical transition to turbulence via perturbations.

The largest piece of future work that is necessary for the continuation of this experiment is automation. Automation was unable to be achieved as the experiment is still in a somewhat embryonic stage. By automating the stepper motors and perturbation system, perturbations can be introduced to a Taylor-Couette flow repeatedly without the need for a human triggering them. This will allow for much greater quantities of data to be collected and therefore allow better conclusions to be drawn from said data.

An automated flow visualization system will need to be implemented. This will likely involve a webcam that will use a Python program to distinguish turbulence from laminar flows and will further streamline data collection. This system also helps in the removal of requiring a human for qualitative analysis of the experiment.

Lastly, the axial aspect ratio could be further researched by moving the boundary rings in the gap between the cylinders. This will provide further insight into how laminar flows react to perturbations under varying system sizes.

5 Conclusion

This experiment set out to find a relation between the minimum perturbation amplitude required to induce turbulence in Taylor-Couette flows and the Reynolds number of said flows. The singular point of data found that with a Reynolds number of $Re_0 = 15091.0842$, a perturbation amplitude 37.5 ml/min was the minimum amplitude required to induce a subcritical transition to turbulence. This leads to a path of continuation of this experiment.

The experimental apparatus and software for this experiment were unfortunately not set up enough to collect enough data to produce meaningful results. By constructing the Taylor-Couette system and coding programs that will be able to be utilized by future researchers, this project has created a good basis for future work. By continuing this study, the relation of perturbation amplitude to Reynolds number may be found, and this will further fluid dynamics research and potentially have vast industrial applications.

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