CS 465: Lambda Homework Solutions

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1. *Syntactic abbreviations*

Convert the following lambda terms into fully parenthesized form.

- λxyz. f x (f y y) (f z) ⇒ (λx. (λy. (λz. (((f x) ((f y) y)) (f z)))))
- λxy. f (λx. x) (λgf. g f x) ⇒ (λx. (λy. ((f (λx. x)) (λg. (λf. ((g f) x))))))

Write these in a minimal form.

- λa. (λb. (a ((b b) a)) ⇒ λab. a (b b a)
- (((λp. (λx. (p x)) (p a)) (λq.q)) (λy.b)) a ⇒ (λp. (λx. p x) (p a)) (λq. q) (λy. b) a

2. *Variables and binding*

Convert the following terms so that no free or bound variables clash.

- (λx. (λxy. x (y x)) y) (λy. (λy. y x) y x)

  ⇒ (λa. (λbc. b (c b)) y) (λd. (λe. e x) d x)

- b a (λab. a (λa. b) b) a (λb. b a)

  ⇒ b a (λxy. x (λz. y) y) a (λw. w a)

3. *Substitution and reduction*

Reduce the following lambda terms to normal form.

- (λfgx. f (g x)) (λy.y) (λfx. f (f x)) x

  → (λgx. (λy. y) (g x)) (λfx. f (f x)) x

  → (λx. (λy. y) ((λfx. f (f x)) x)) x

  → (λy. y) ((λfx. f (f x)) x)

  → (λfx. f (f x)) x

  → λa. x (x a)

- (λfx. f (f x)) (λy. c (y y)) b

  → (λx. (λy. c (y y)) ((λy. c (y y)) x)) b

  → (λy. c (y y)) ((λy. c (y y)) b)

  → c ((λy. c (y y)) b ((λy. c (y y)) b))

  → c (c (b b) ((λy. c (y y)) b))

  → c (c (b b) (c (b b)))