$A \times B$

"For *some* A, a B."

Multiplication is like **repeated addition**. What can this mean in **grids**?

Given (A × B), we have A repetitions of B added together: (B + B + …).

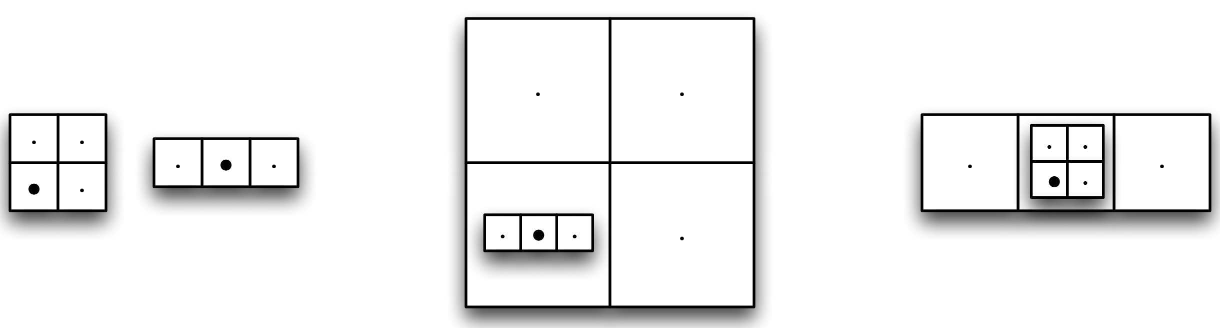
Except A is a *grid,* not a number. So instead, maybe |A| repetitions of B, where |A| is the *cardinality* (or size) of the type A. In other words, we flatten or "unravel" the type to a simple number, and use that as the number of repetitions.

$$\overbrace{(B + B + \ldots + B)}^{|A| \text{ times}}$$

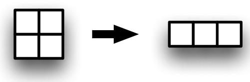
Now, what information do we have when we have a value of this type? Since all the summands (the Bs) are the same, a "tagged choice" of a B value is really just a "tag", a number k<n, paired with a value of type B. That is to say, in some sense, a value of type (|A| × B).

But now imagine that we "re-ravel" that k value back into the "shape" of the type A, so it is not just a flat number, but a location in a(n A-shaped) structure. Now it is clear that we have just an A value and a B value, two locations in two grids. But really they are jointly the location in a grid of "intersections", where we have, orthogonally, a paired location for very distinct combinations of locations from A and B. (Likewise, a value of a labeled sum of Bs, for some label-set that might as well be a symbolic alphabet type A, is just a pair of an A "tag" with a B value.)

But … note that we could imagine the B value as being contained in the A grid at that particular location … or *vice versa,* by commutativity. So maybe × is even less commutative than we thought: maybe (A × B) is a B located in an A-shaped grid, whereas (B × A) is an A located in a B-shaped grid. In the end, these would b isomorphic, not equal, but then iso is all we really need … .



$$A \to B \quad \equiv \quad B^{A}$$



"For *every* A, a B."

Exponentiation is like **repeated multiplication**. What can this mean in **grids**?

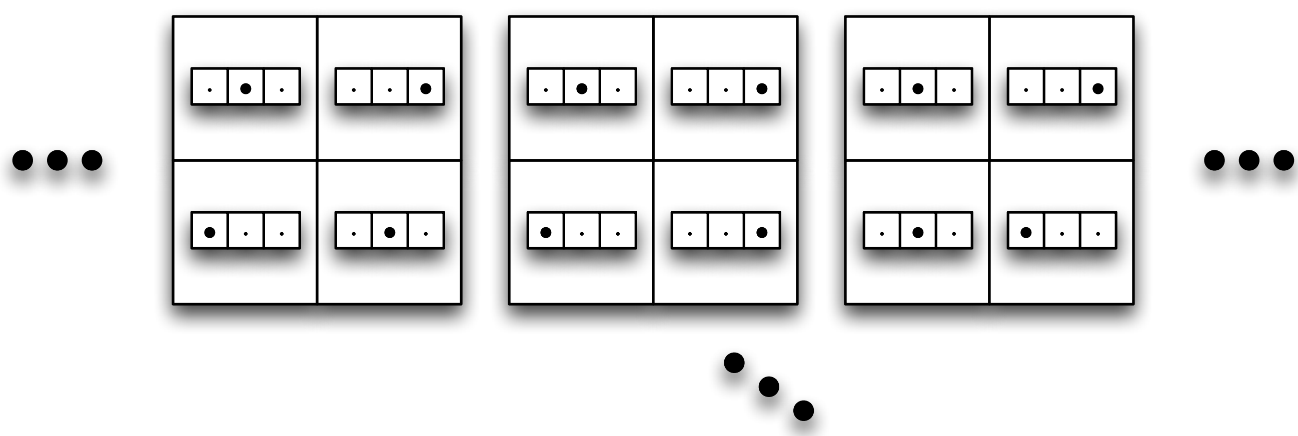
Given $(A \to B) \equiv B^{A}$, we have A repetitions of B multiplied together: $(B \times B \times \ldots)$.

Except A is a *grid,* not a number. So instead, maybe |A| repetitions of B, where |A| is the cardinality or size of the type A. In other words, we flatten or "unravel" the type to a simple numeral, and use that as the number of repetitions.

$$\overbrace{(B \times B \times \ldots \times B)}^{|A| \text{ times}}$$

Now, what information do we have when we have a value of this type? Since all the multiplicand (the Bs) are the same, a "vector" of B values of length k is really just a B value for *every* k<n such that n = |A|. That is to say, in some sense, a value of type (|A| → B). (Likewise, a value of a labeled product of Bs, for some label-set that might as well be a symbolic alphabet type A, is just a finite function from an A "label" into B values.)

But now imagine that we "re-ravel" that k value back into the "shape" of the type A, so it is not just a flat number, but a location in a(n A-shaped) structure. Now it is clear that we have just an A-shaped grid full of B values, or a function from A to B. But really they are jointly the location in a grid of "… *<somethings?>* …", where we have, orthogonally, a single location for every distinct combination of B values in all A locations.



(Really, these should be arranged in something like a 4-dimensional shape, with each "inner choice" varying along an axis of length 3, but all axes orthogonal. I.e., 81 such squares, arranged in something like a 4-D Rubik's cube … .)