

## Rules for natural deduction proofs

*formulas*  $P ::= \text{false} \mid V \mid \neg P \mid P \vee P \mid P \wedge P \mid P \Rightarrow P$

*variables*  $V ::= a \mid b \mid \dots$

$$\frac{P \vee Q \quad \boxed{\begin{array}{c} P \\ \vdots \\ R \end{array}} \quad \boxed{\begin{array}{c} Q \\ \vdots \\ R \end{array}}}{R} \quad [\vee E]$$

$$\frac{P}{P \vee Q} \quad [\vee I]$$

$$\frac{Q}{P \vee Q} \quad [\vee I]$$

$$\frac{P \wedge Q}{P} \quad [\wedge E]$$

$$\frac{P \wedge Q}{Q} \quad [\wedge E]$$

$$\frac{P \quad Q}{P \wedge Q} \quad [\wedge I]$$

$$\frac{P \Rightarrow Q \quad P}{Q} \quad [\Rightarrow E]$$

$$\frac{\boxed{\begin{array}{c} P \\ \vdots \\ Q \end{array}}}{P \Rightarrow Q} \quad [\Rightarrow I]$$

$$\frac{P \quad \neg P}{\text{false}} \quad [\neg E]$$

$$\frac{\boxed{\begin{array}{c} P \\ \vdots \\ \text{false} \end{array}}}{\neg P} \quad [\neg I]$$

$$\frac{\text{false}}{P} \quad [\text{false } E]$$

$$\frac{\neg \neg P}{P} \quad [\neg \neg E]$$