## Rules for sequent-style natural deduction

*formulas* $P ::= \mathit{false} \mid V \mid \neg P \mid P \vee P \mid P \wedge P \mid P \Rightarrow P$

*variables* $V ::= a \mid b \mid \ldots$

*contexts* $\Gamma ::= \Gamma, P \mid \mathit{empty}$

*sequents* $\Gamma \vdash P$

$$\frac{\Gamma \vdash P \vee Q \qquad \Gamma, P \vdash R \qquad \Gamma, Q \vdash R}{\Gamma \vdash R}\ [\vee E] \qquad \frac{\Gamma \vdash P}{\Gamma \vdash P \vee Q}\ [\vee IL] \qquad \frac{\Gamma \vdash Q}{\Gamma \vdash P \vee Q}\ [\vee IR]$$

$$\frac{\Gamma \vdash P \wedge Q}{\Gamma \vdash P}\ [\wedge EL] \qquad \frac{\Gamma \vdash P \wedge Q}{\Gamma \vdash Q}\ [\wedge ER] \qquad \frac{\Gamma \vdash P \qquad \Gamma \vdash Q}{\Gamma \vdash P \wedge Q}\ [\wedge I]$$

$$\frac{\Gamma \vdash P \Rightarrow Q \qquad \Gamma \vdash P}{\Gamma \vdash Q}\ [\Rightarrow E] \qquad \frac{\Gamma, P \vdash Q}{\Gamma \vdash P \Rightarrow Q}\ [\Rightarrow I]$$

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$$\frac{\Gamma \vdash P \qquad \Gamma \vdash \neg P}{\Gamma \vdash \mathit{false}}\ [\neg E] \qquad \frac{\Gamma, P \vdash \mathit{false}}{\Gamma \vdash \neg P}\ [\neg I]$$

$$\frac{\Gamma \vdash \mathit{false}}{\Gamma \vdash P}\ [\mathit{false}\ E] \qquad \frac{\Gamma \vdash \neg \neg P}{\Gamma \vdash P}\ [\neg\neg E]$$

$$\frac{}{\ldots, P, \ldots \vdash P}\ [ID]$$