Rules for first-order logic (sequent-style natural deduction)

terms	$T ::= X \mid F^n(T_1, \ldots, T_n)$
formulas	$P ::= \ldots \mid R^n(T_1, \ldots, T_n) \mid P \vee P \mid P \wedge P \mid P \Rightarrow P \mid \forall X.\, P[X] \mid \exists X.\, P[X]$
variables	$X ::= x \mid y \mid \ldots$
function symbols	$F ::= f^n \mid g^m \mid \ldots$ *(ranked)*
contexts	$\mathbf{\Gamma} ::= \mathbf{\Gamma}, P \mid$ *empty*
relation symbols	$R ::= p^n \mid q^m \mid \ldots$ *(ranked)*
sequents	$\mathbf{\Gamma} \vdash P$

$$\frac{\mathbf{\Gamma} \vdash P \vee Q \qquad \mathbf{\Gamma}, P \vdash R \qquad \mathbf{\Gamma}, Q \vdash R}{\mathbf{\Gamma} \vdash R}\ [\vee E] \qquad \frac{\mathbf{\Gamma} \vdash P}{\mathbf{\Gamma} \vdash P \vee Q}\ [\vee IL] \qquad \frac{\mathbf{\Gamma} \vdash Q}{\mathbf{\Gamma} \vdash P \vee Q}\ [\vee IR]$$

$$\frac{\mathbf{\Gamma} \vdash P \wedge Q}{\mathbf{\Gamma} \vdash P}\ [\wedge EL] \qquad \frac{\mathbf{\Gamma} \vdash P \wedge Q}{\mathbf{\Gamma} \vdash Q}\ [\wedge ER] \qquad \frac{\mathbf{\Gamma} \vdash P \qquad \mathbf{\Gamma} \vdash Q}{\mathbf{\Gamma} \vdash P \wedge Q}\ [\wedge I]$$

$$\frac{\mathbf{\Gamma} \vdash P \Rightarrow Q \qquad \mathbf{\Gamma} \vdash P}{\mathbf{\Gamma} \vdash Q}\ [\Rightarrow E] \qquad \frac{\mathbf{\Gamma}, P \vdash Q}{\mathbf{\Gamma} \vdash P \Rightarrow Q}\ [\Rightarrow I]$$

(plus rules for false *and negation, if you like that sort of thing)*

$$\frac{\mathbf{\Gamma} \vdash \forall x.\, P[x]}{\mathbf{\Gamma} \vdash P[T]}\ [\forall E] \qquad y \notin FV(\mathbf{\Gamma}) \cup FV(P[x]) \quad \frac{\mathbf{\Gamma} \vdash P[y]}{\mathbf{\Gamma} \vdash \forall x.\, P[x]}\ [\forall I]$$

$$\frac{\mathbf{\Gamma} \vdash \exists X.\, P[X] \qquad \mathbf{\Gamma}, P[y] \vdash Q}{\mathbf{\Gamma} \vdash Q}\ [\exists E] \qquad \frac{\mathbf{\Gamma} \vdash P[T]}{\mathbf{\Gamma} \vdash \exists X.\, P[X]}\ [\exists I]$$

$y \notin FV(\mathbf{\Gamma}) \cup FV(P[x]) \cup FV(Q)$

$$\frac{}{\mathbf{\Gamma}, P \vdash P}\ [ID]$$