

A sequent-style natural deduction proof of the following proposition:

1

$$(P \wedge (P \Rightarrow Q)) \Rightarrow (P \wedge Q)$$

The proof:

$$\begin{array}{c}
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge (P \Rightarrow Q)} [ID] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge (P \Rightarrow Q)} [\wedge EL] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P} [\wedge EL] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge (P \Rightarrow Q)} [ID] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge (P \Rightarrow Q)} [\wedge ER] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \Rightarrow Q} [\wedge ER] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P} [ID] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P} [\wedge EL] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P} [\Rightarrow E] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash Q} [\wedge I] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge Q} [\Rightarrow I] \\
 \frac{}{\vdash (P \wedge (P \Rightarrow Q)) \Rightarrow (P \wedge Q)} [\Rightarrow I]
 \end{array}$$

Two sequent-style natural deduction proofs of the following proposition:

2

$$(P \wedge (P \Rightarrow Q)) \Rightarrow (P \vee Q)$$

One proof (P):

Another proof (Q):

$$\begin{array}{c}
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge (P \Rightarrow Q)} [ID] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge (P \Rightarrow Q)} [\wedge EL] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P} [\wedge EL] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \vee Q} [\vee I] \\
 \frac{}{\vdash (P \wedge (P \Rightarrow Q)) \Rightarrow (P \vee Q)} [\Rightarrow I]
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge (P \Rightarrow Q)} [ID] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \wedge (P \Rightarrow Q)} [\wedge ER] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \Rightarrow Q} [\wedge ER] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P} [ID] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P} [\wedge EL] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P} [\Rightarrow E] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash Q} [\vee I] \\
 \frac{}{P \wedge (P \Rightarrow Q) \vdash P \vee Q} [\vee I] \\
 \frac{}{\vdash (P \wedge (P \Rightarrow Q)) \Rightarrow (P \vee Q)} [\Rightarrow I]
 \end{array}$$