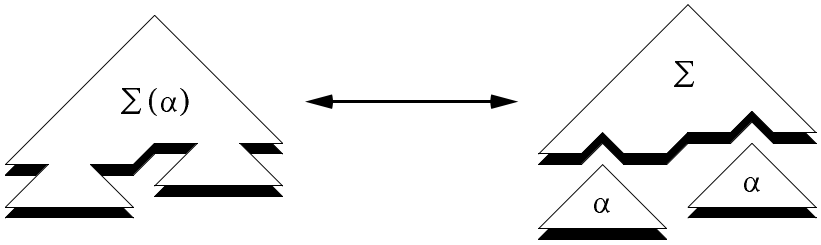
Analytical And Structural Polymorphism Expressed Using Patterns Over Types

Karl Fritz Ruehr



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Dedicated to the memory of Joanne Wagner

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