Summations

Summations can be finite or infinite. We define infinite infinite sums as

$$\sum_{k=1}^{\infty} f(k) = \lim_{n\to\infty} \sum_{k=1}^{n} f(k)$$

If we have a sum with asymptotic notation:

$$\sum_{k=1}^{n} \Theta(f(k)) = \Theta(\sum_{k=1}^{n} f(k))$$

Note that on left, Θ is on k, on right Θ is on n.

Known Series

- Arithmetic series
  This came up in insertion sort.

  $$\sum_{k=1}^{n} k = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

  Which is $\Theta(n^2)$.

- Geometric series:

  1. Finite sum
     $$\sum_{k=0}^{n} x^k = 1 + x + x^2 + ... + x^n = \frac{x^{n+1} - 1}{x - 1}$$
  2. Infinite sum
     $$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + ... = \frac{1}{x - 1}$$

     for $0 \leq x < 1$

- harmonic series (infinite sum blows up)

  $$\sum_{k=0}^{\infty} \frac{1}{k} = \infty$$

  $$\sum_{k=0}^{n} \frac{1}{k} = \ln n + O(1)$$

- Lots of sums are available in tables of mathematical functions.