Name: _____

CS443: Analysis of Algorithms Final

Score:	1. (max 30)	3. (max 20)
	2. (max 20)	4. (max 25)
	Total: (max 95)	

This exam is closed book. Calculators are not allowed.

1. (25 pts total) $O(n)$ (big-Oh) Complexity

   (a) (5 pts) State the definition of $O(n)$.

   (b) (10 pts) Consider the recurrence where $T(1) = 1$ and $T(n) = 2T(n/2) + n$ for all $n > 1$. Give a careful inductive proof that this is $O(nlgn)$. You may restrict the proof to the case where n is a power of 2.

(c) (10 pts) *Formally* prove the following theorem: If $f(n) = O(s(n))$ and $h(n) = O(r(n))$ then $f(n) + h(n) = O(s(n) + r(n))$.

(d) (5 pts) The above theorem is used in the following "justification" that the Fibonacci function is $O(n)$:

i. Base case ($n \leq 2$): $F(1) = 1$, which is $O(1)$ and $F(2) = 2$, which is $O(2)$.

ii. Inductive step ($n > 2$): Assume the claim is true for $n' < n$. Consider n: $F(n) = F(n-1) + F(n-2)$. By induction, $F(n-1)$ is $O(n-1)$ and $F(n-2)$ is $O(n-2)$. Then by the above theorem, $F(n)$ must be $O((n-1)+(n-2))$. Therefore $F(n)$ is $O(n)$ since $O((n-1)+(n-2))$ is $O(n)$.

What is wrong with this "justification".

2. (20 pts) Sorting: Compare and contrast the following
   (a) Heapsort
   (b) Quicksort
   (c) Radix Sort

3. (20 pts total) Professor Midas drives his fuel efficient Honda Insight hybrid car from Portland to San Francisco along Interstate 5. His car can go n miles on a full tank of gas. His map gives the distances between gas stations on his route.

   (a) (5 pts) Describe an optimal greedy algorithm by which Professor Midas can determine at which gas stations he should stop.

   (b) (5 pts) State the Optimal Substructure Property.

   (c) (10 pts) Prove that your algorithm satisfies the Optimal Substructure Property. Begin your proof by assuming that $A = \{s_1, s_2, ..., s_m\}$ is an optimal solution and that s_1 is the greedy choice.

4. (25 pts total) Professor Peabody is making a trip that is similar to Midas's. Like Midas, his gas tank has a range of n miles. However, his bladder has a range of n' miles, and he is fussy about using the bushes. He has a map of pit stops along the route. Some of them are roadside rest areas where they have rest rooms but no gas, and others are gas stations, but the rest rooms are out of order at some of the gas stations. His map details all of this information.

   (a) (10 pts) Describe a greedy solution to this problem.

   (b) (5 pts) Is this greedy solution optimal? Why or why not.

   (c) (10 pts) *Sketch* out a possible dynamic programming approach to this problem.