Frames and Homogeneous Coordinates

Coordinate systems and frames:

When we write a point as P = $\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix}$. What does this mean?

We assume that there are coordinate axes defined by a set of basis vectors and that α, β, γ are coefficients of these vectors v_1, v_2, v_3.

$$P = \alpha v_1 + \beta v_2 + \gamma v_3 = \begin{pmatrix}\alpha & \beta & \gamma\end{pmatrix}\begin{pmatrix}\vec{v}_1\\ \vec{v}_2\\ \vec{v}_3\end{pmatrix}$$

Typically we have $v_1 = (1,0,0)^T$ $v_2 = (0,1,0)^T$ $v_3 = (0,0,1)^T$

However, vectors have no location so what does it mean for our axes to be vectors? To be precise, we need to specify point to serve as an origin as well.

Frame = point + 3 basis vectors

$P = P0 + \alpha v_1 + \beta v_2 + \gamma v_3$ =

$$\begin{pmatrix}\alpha & \beta & \gamma & 1\end{pmatrix}\begin{pmatrix}v_1\\ v_2\\ v_3\\ P_0\end{pmatrix} = 1\begin{pmatrix}P_{0x}\\ P_{0y}\\ P_{0z}\end{pmatrix} + \alpha\begin{pmatrix}v_{1x}\\ v_{1y}\\ v_{1z}\end{pmatrix} + \beta\begin{pmatrix}v_{2x}\\ v_{2y}\\ v_{2z}\end{pmatrix} + \gamma\begin{pmatrix}v_{3x}\\ v_{3y}\\ v_{3z}\end{pmatrix}$$

This is referred to as homogeneous coordinates.

Note that a vector has no location, so to represent a vector in h-coords becomes

$$w = \alpha v_1 + \beta v_2 + \gamma v_3 = \begin{pmatrix}\alpha & \beta & \gamma & 0\end{pmatrix}\begin{pmatrix}v_1\\ v_2\\ v_3\\ P_0\end{pmatrix}$$

Thus, we now write all points as $\begin{pmatrix}x\\ y\\ z\\ 1\end{pmatrix}$ and all vectors as $\begin{pmatrix}x\\ y\\ z\\ 0\end{pmatrix}$ with the understanding that these represent coefficients in some basis set (v_1, v_2, v_3, P0).

This has the nice quality that now points and vectors have a different representation; One can easily identity which is which.