Name:

| Score: | 1. | $(\max 10)$ | 4. | $(\max 15)$ |
|--------|--------|-------------|-----|-------------|
| | 2. | $(\max 15)$ | 5. | $(\max 15)$ |
| | 3. | $(\max 5)$ | 6. | $(\max 23)$ |
| | Total: | (max | 83) | (percent) |

CS-445 Midterm Fall 2000

1. (5pts each, 10 pts total) Give the definitions of the following terms. You *must* use complete English sentences. Be as specific as possible.

(a) aliasing (in signal processing). Include a picture.

(b) tri-stimulus model

- 2. (5 pts each, 15 pts total) The perceptual qualities of a light source can be described using the three quantities: hue, saturation, and lightness. Each of these three properties also corresponds to properties of the energy spectrum.
 - (a) Hue corresponds to the mean value of the energy spectrum (see picture below). Perceptually, what does hue correspond to? That is, what looks different when you look at light sources with different hues?



(b) If hue corresponds to mean, what property of the energy spectrum does saturation correspond to? That is, if two light sources have the same hue and lightness but different saturation, how do their energy spectra differ? Include a picture.

(c) What property of the energy spectrum does lightness correspond to? That is, if two light sources have the same hue and saturation but different lightness, how do their energy spectra differ? Include a picture. 3. (5 pts each, 15 pts total) 2D Transforms: Write down the 3x3 matrix transform (or sequence of matrix transforms) needed to perform each of the following transformations. Use homogeneous coordinates. If more than one matrix is needed make sure the order of multiplication is clear. You do not need to multiply the matrices out. You also do not need to numerically evaluate the trig functions (e.g. sin, cos).

When more than one matrix is required, write down in words what each matrix individually represents.

(a) Project onto the x-axis.

(b) Rotate by 35° about the point (-2,4).

(c) Scale by 5 along a 15° angle relative to the point (-3,7).

4. (5 pts each, 15 pts total) 3D Transforms: Write down the 4x4 matrix transform (or sequence of matrix transforms) needed to perform each of the following transformations. Use homogeneous coordinates. If more than one matrix is needed make sure the order of multiplication is clear. You do not need to multiply the matrices out. You also do not need to numerically evaluate the trig functions (e.g. sin, cos).

When more than one matrix is required, write down in words what each matrix individually represents.

(a) Scale by 5 along the y axis.

(b) Rotate by 35° about the line defined by x=1, y=3.

(c) Rotate by 25° about the vector (0,1,1).

5. (5 pts): What are the CIE's XYZ colors and why were they developed?

Do one of the following two problems. Please circle the one you want graded. If you don't specify which one then I will choose randomly. Write your answers *after* the full problem statement in the space provided. Please make sure each part is clearly labeled.

6. (23 pts total) Parametric Line Clipping.

Given the two points P_0 and P_1 that define an edge, the *parametric vector equation* of the line connecting them is

$$P = (P_1 - P_0)t + P_0$$

- (a) (2 pts) Suppose $P_0 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $P_1 = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$. Use the above equation to find the parametric equation for the line that connects them.
- (b) (5 pts) Let Q be a specific point on a clip line and N be the normal. Let P be an arbitrary point on the line. What is a simple vector equation that relates P, Q, and N? (hint: use dot products)
- (c) (9 pts) Use the parametric vector equation given in the beginning of this problem and use the result from part (b) to derive a general equation for the intersection of the edge and the clip line. (hint: Plug one equation into the other and solve for t.)
- (d) (5 pts) Assume $Q = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, $N = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and that P_0 and P_1 are given in (a). Use the equation derived derived in (d) to compute the value of t at the intersection point.
- (e) (2 pts) What are the values of x and y at this intersection point?
- (f) (0 pts) Suggestion: Draw a picture of all of this to make sure your answer sort of makes sense.

- 7. (23 pts total) Suppose your head is at the origin oriented so that you are looking along the vector (1,1,2). Assume that your shoulders are level (i.e parallel to the x-y plane). You want to rotate your head so that it is looking down the x-axis.
 - (a) (7 pts) What is the sequence of rotations about the major axes that will do this. Explain your steps.
 - (b) (7 pts) Use the method of special orthogonal matrices to obtain the complete rotation matrix. Explain your steps.
 - (c) Suppose you not only want your head to look down the x-axis but you also want your shoulders to remain level (i.e. if you stretch out your left arm it should be in the x-y plane before and after the rotation.)



- i. (1 pt) Does your answer in (a) do this? Explain.
- ii. (1 pt) Does your answer in (b) do this? Explain.
- iii. (7 pts) Suppose you want your left arm to end up pointing along the positive z-axis after the rotation (while your head is looking down the x-axis). How would you modify the steps in (b) to insure that this will happen?