October 14, 2010 Name ___________________

SOLUTIONS

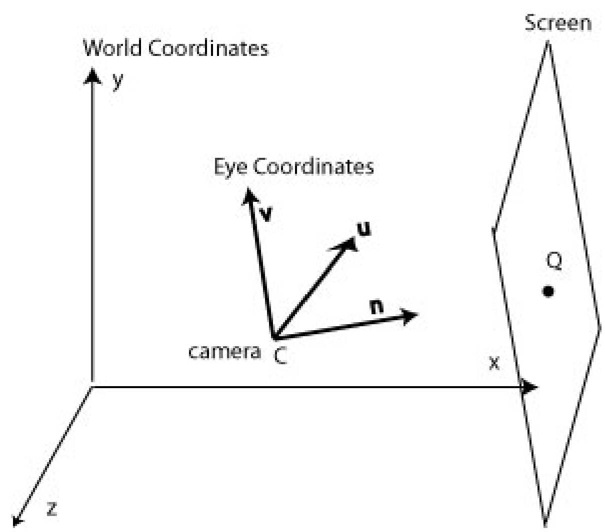
CS445 Midterm

Fall 2010

1.	(max = 10)	5.	(max = 6)
2.	(max = 18)	6.	(max = 12)
3.	(max = 8)	7.	(max = 10)
4.	(max = 16)	8.	(max = 20)
Final Score _________(max=100)			

1) (10 pts) Given the following inputs:
   - C = camera location
   - Q = location of the center of the screen (i.e. the camera look-at point)
   - VUP = the up direction vector

   How does one calculate the normalized eye coordinate basis vectors, u,v,n, (see picture). Assume we are using a *right handed* coordinate system as shown. Show your work.



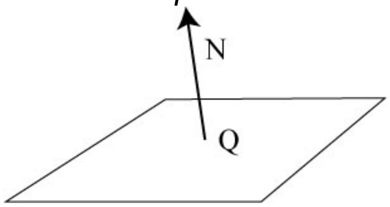
$$n = \frac{(Q - C)}{\|Q - C\|}$$

$$u = \frac{VUP \times n}{\|VUP \times n\|}$$

$$v = n \times u$$

2) (6 pts each, 18 pts total)

a) For a plane defined by a normal N and a point Q, how does one calculate the point of intersection of the plane with a ray defined by $P = P_0 + t\,dir$. That is, what is *both the t value at the intersection point and the actual coordinates of the point p(x,y,z) itself*. Show your work.



$$(P - Q) \bullet N = 0$$

$$(P_0 + t\,dir - Q) \bullet N = 0$$

$$t = \frac{(Q - P_0) \bullet N}{dir \bullet N}$$

$$p(x,y,z) = P_0 + t\,dir = P_0 + \frac{(Q - P_0) \bullet N}{dir \bullet N}\,dir$$

b) Show the calculation for both t and p(x,y,z) given :

$P_0 = (1,1,2)$, $N = (2, -1,1)$, $dir = (-2,1,1)$, $Q = (1,4,3)$.

$$t = \frac{((1,4,3) - (1,1,2)) \bullet (2,-1,1)}{(-2,1,1) \bullet (2,-1,1)} = \frac{(0, 3, 1) \bullet (2, -1,1)}{-4 -1 + 1} = \frac{0 - 3 + 1}{-4} = \frac{-2}{-4} = 1/2$$

$$p(x,y,z) = (1,1,2) + \tfrac{1}{2}\,(-2,1,1) = (1,1,2) + (-1, \tfrac{1}{2}, \tfrac{1}{2}) = (0, 1.5, 2.5)$$

c) How do you tell if the ray is parallel to the plane but not lying in the plane?

The ray is parallel if $dir \bullet N = 0$.

If the ray is lying in the plane then P_0 must be in the plane so that

$(Q - P_0) \bullet N = 0$ as well

3) (2 pts each, 8 pts total) Given the vectors: v1 = (1, 0, 0), v2 = (-1, 0, 0), v3 = (0,0,1), v4 = (0,1,0) Evaluate the following where the symbol • indicates the dot product and × indicates the cross product. (You do not need to explicitly calculate the cross product, but instead, you just need to reason about it)

a) v1 × v1 = (1, 0, 0) × (1, 0, 0) = (0,0,0)

b) v1 × v2 = (1, 0, 0) × (-1, 0, 0) = (0,0,0)

c) v1 × v3 = (1, 0, 0) × (0,0,1) = (0, -1, 0)

d) v1 × v4 = (1, 0, 0) × (0,1,0) = (0,0,1)

4) (4 pts each, 16 pts total) 2D Transforms: What is the 3x3 matrix transform (in homogeneous coordinates) for the 2D transformations below. Also give the inverse.

a) Scale by s_x in the x direction:

The transform:

$$\begin{pmatrix} sx & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The inverse:

$$\begin{pmatrix} 1/sx & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b) A negative rotation of θ degrees:

The transform:

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The inverse:

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

c) A reflection about the y-axis.

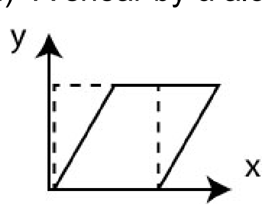
The transform:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The inverse:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

d) A shear by α along the x axis



$$\begin{pmatrix} 1 & \alpha & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The inverse: $$\begin{pmatrix} 1 & -\alpha & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5) (6 pts total) Determinants:

a) (2 pts) What does the determinant tell you about the transform?

The change in volume of a shape when it is transformed.

Note, if the determinant is zero then that means that the shape has been collapsed and information has been lost => there is no inverse.

b) (4 pts) What is the determinant for the following transformations:

i) Uniform scale by 3. 3x3 = 9

ii) A rotation of 68 degrees: 1 since a rotation does not change the volume.

6) (6 pts each, 12 pts total) *2D Transforms*: Give the *sequence* of matrix transformations (order matters!) that is required to do the following transformations. You do not need to give the actual matrix. Instead, identify each matrix using the syntax $S(s_x, s_y)$ for a scale, $R(\theta)$ for a rotation, $T(t_x, t_y)$ for a translation. For each, also give the *sequence* of transformations needed to obtain the inverse.

a) Scale uniformly by 5 about the point at (-1,10).

T(-1,10) S(5,5) T(1,-10)

Inverse:

T(-1,10) S(1/5,1/5) T(1,-10)

b) Scale by 5 along a positive 45 degree angle about the point (4,9)

T(4,9) R(45) S(5,1) R(-45) T(-4,-9)

Inverse:

T(4,9) R(45) S(1/5,1) R(-45) T(-4,-9)

(5 pts each, 10 pts total) In the graphics pipeline:

c) What are Model Coordinates? Why are they used?

In Model Coordinates are selected so that an object is oriented and located in a way that is "natural", i.e. where specifying the vertices is simplest and where the origin is the typical center of rotation or scale. For example, a cube is modeled in a coordinate system whose axes are parallel to the sides of the cube, thus vertices are easy to identify. It's center is placed at the origin because this is the typical center point for scaling and rotation. When such a cube is scaled, it remains centered at the origin. A cone in modeled so that its axis is centered about and aligned with a coordinate axis.

Thus, Model Coordinates are used because they make the modeling and transformation of objects much easier!

d) What are Eye Coordinates? Why are they used?

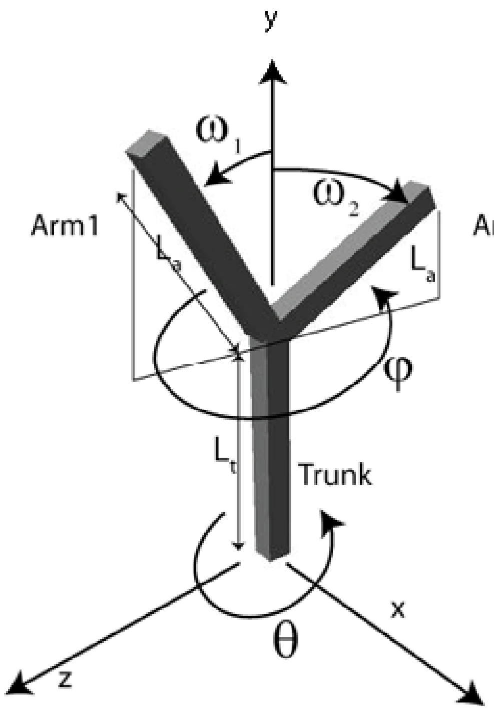
The Eye Coordinates is the coordinate system where

1) the camera is sitting at the origin
2) the camera is looking directly down the positive or negative z-axis depending on whether a right or left handed coordinate system is used
3) the y-axis is up relative to camera.
4) the x-axis is to the right relative to camera.

This is done so that it is easy to

1) project down to 2D and
2) determine the distance between an object and the camera (z buffer)

7) (20 pts total) Below is an adjustable rack used to hang and display items in a store. The trunk can be 1) translated about the floor and 2) rotated about a vertical axis (θ) about its center. Attached to the trunk are two arms which rotate *together* about a vertical axis (φ). However, they can also independently rotate up and down by angles $\omega 1$ and $\omega 2$. The trunk has length L_t and the arms are both of length L_a. Assume the unlabeled dimensions are of length 1.



Draw the <u>scene graph</u> for the rack. Be sure to include all transformations needed to scale, move and place each part properly in the scene.

For the trunk and arms, assume you start with a cube, e.g. the glut cube class which draws a unit cube centered at the origin.

Scale transformations should be indicated as `scale(sx,sy,sz)` where you fill in specific values for `sx`, `sy`, and `sz`.
Similarly, translations and rotations should have the form `translate(tx,ty,tz)` and `rotate(angle, axis)`, respectively.
Indicate push/pops where needed.

Note, there are slight variations of this graph that are also correct.