

## CS445 Midterm

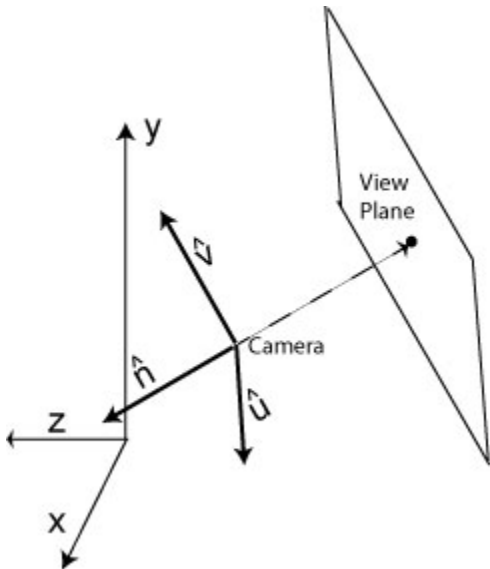
Fall 2012

1. (max = 12)	4. (max = 25)
2. (max = 15)	5. (max = 18)
3. (max = 10)	6. (max = 20)
Final Score: (max=100)	

1) (12 pts) Ray Tracing: Suppose you are given:

- VPN = a vector that points in a direction *opposite* the way the camera looks
- VUP = the up direction vector

How does one calculate the normalized eye coordinate basis vectors:  $\hat{u}$ ,  $\hat{v}$ ,  $\hat{n}$  (see picture). Assume you are using a right handed coordinate system as shown.



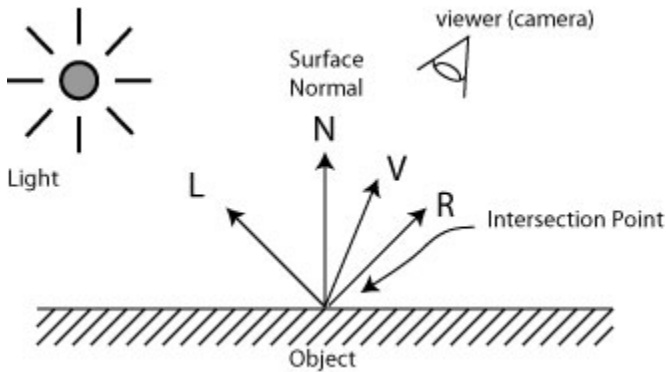
$$\hat{u} = \frac{VUP \times \hat{n}}{\|VUP \times \hat{n}\|}$$

$$\hat{v} = \hat{n} \times \hat{u}$$

$$\hat{n} = \frac{VPN}{\|VPN\|}$$

2) **Phong Lighting** (5 pts each, 15 pts total)

Suppose you are given the parameters as shown in the picture:  $L$  (unit vector in direction of light),  $R$  (unit vector in direction of reflected light),  $N$  (unit normal), and  $V$  (unit vector in direction of viewer). You are also given the reflection coefficients  $k_d$  and  $k_s$ , the specularity  $n$ , the surface color  $C_{surf}$  (which is the same for both diffuse and specular), and light color  $C_{Light}$ .



a) What is  $R$  in terms of  $L$ ,  $N$ , and  $V$ ?

$$R = 2 (N \cdot L) N - L$$

b) What is the color contribution of diffuse light to the pixel color at the intersection point?

$$\text{Diffuse color} = k_d \cdot C_{surf} \cdot C_{Light} (N \cdot L)$$

c) What is the color contribution of specular light to the pixel color at the intersection point?

$$\text{specular color} = k_s \cdot C_{Light} (V \cdot R)^n$$

3) (5 pts each, 10 pts total) Rays

- a) What is the parametric equation of a ray? Besides giving the formula, please explain in words what each of the terms in the formula represents. Include a picture.

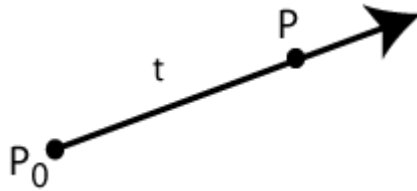
Points P on a ray must satisfy:  $P = P_0 + t \text{ dir}$

Where

$P_0$  = the starting point of the ray

dir = vector pointing along the ray direction

t = positive scalar parameter indicating the distance P is along the ray



- b) Given an arbitrary point Q, explain (using words and equations) how you determine if Q is a point on the ray.

A point Q is on the ray if it satisfies the equation  $Q = P_0 + t \text{ dir}$  for some positive value of t.

Writing it another way, we have

$$(Q - P_0) - t \text{ dir} = 0$$

This is a vector equation which says that dir must be parallel to  $(Q - P_0)$ .

There are several ways to check for this.

If we define  $w = Q - P_0$ , then we must satisfy:

$$(w_x - t \text{ dir}_x, w_y - t \text{ dir}_y, w_z - t \text{ dir}_z) = (0,0,0)$$

Or, there must exist a single positive t such that

$$t = w_x / \text{dir}_x = w_y / \text{dir}_y = w_z / \text{dir}_z$$

If no such t exists, then Q is not on the ray.

4) (5 pts each, 25 pts total) **3D Transforms:** What is the 4x4 matrix transform (in homogeneous coordinates) for the 3D transformations below. *Also give the inverse.*

a) Scale by 5 in the z direction:

The transform:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The inverse:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) A rotation of 10 degrees about the x axis:

The transform:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(10) & -\sin(10) & 0 \\ 0 & \sin(10) & \cos(10) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The inverse: replace 10 with -10 to give:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(10) & \sin(10) & 0 \\ 0 & -\sin(10) & \cos(10) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

c) A projection onto the yz-plane.

The transform:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The inverse:

no inverse exists

d) A translation by 10 along x and by -5 along y.

The transform:

$$\begin{pmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The inverse:

$$\begin{pmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

e) A reflection through the xz-plane

The transform:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The inverse:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5) (6 each, 18 pts total) **Composition of 3D Transforms:** What is the sequence of transformations needed to achieve the operations given below. Also, include the corresponding inverse. You do not need to write out the 4x4 matrices. Instead, make use of the syntax:

Scale:  $S(s_x, s_y, s_z)$

Translation:  $T(t_x, t_y, t_z)$

Rotation:  $R_x(\Theta), R_y(\Theta), R_z(\Theta)$ .

a) A rotation of 20 degrees about an axis that goes through the point (a,b,c) and is parallel to the y axis.

The transforms:

$$T(a,b,c) R_y (20) T(-a,-b,-c)$$

The inverse:

$$T(a,b,c) R_y (-20) T(-a,-b,-c)$$

b) A scale by 5 (with fixed point at the origin) along the direction defined by the line from (0,0,0) to (-1, 0, 1).

The transforms:

$$R_y (45) S(5,1,1) R_y (-45) \quad \text{or} \quad R_y (-45) S(1,1,5) R_y (45)$$

The inverse:

$$R_y (45) S(1/5,1,1) R_y (-45) \quad \text{or} \quad R_y (-45) S(1,1,1/5) R_y (45)$$

c) A scale by 2 with fixed point (2,3,4) and along the direction parallel to the x axis.

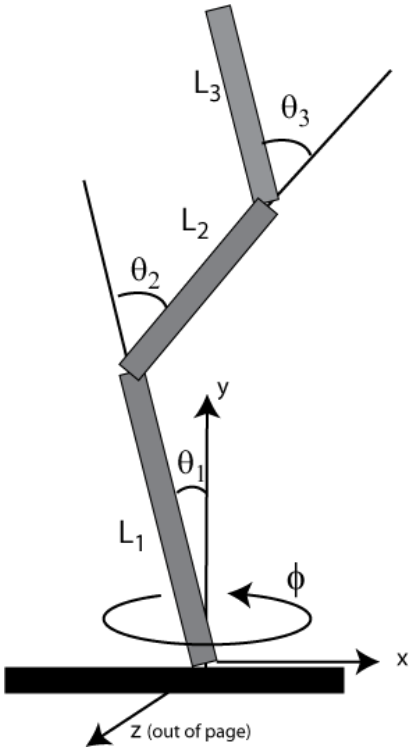
The transforms:

$$T(2,3,4) S(2,1,1) T(-2,-3,-4)$$

The inverse:

$$T(2,3,4) S(1/2,1,1) T(-2,-3,-4)$$

6) (20 pts) **Scene Graphs:** Below is a picture of a 3 segment robotic arm sitting on a base. Each segment is a cylinder of radius  $r$  and length  $L_i$ , with  $i=1,2, or  $3$ . The arm segments can be rotated as shown.$



Draw the scene graph for the robotic arm (not including the black base).

Assume that you have access to a cylinder primitive that has radius 1, height 1, is centered at the origin, and aligned with the z-axis.

Be sure to include all transformations. Scale transformations should be indicated as  $S(s_x, s_y, s_z)$  where you fill in specific values for  $s_x, s_y,$  and  $s_z$ . Similarly, translations and rotations should have the form  $T(t_x, t_y, t_z), Rx(\text{angle}), Ry(\text{angle}),$  and  $Rz(\text{angle})$ .

Indicate push/pops where needed.

