Dec 15, 2010 Name ___________________

CS445 Final Exam Solutions

Fall 2010

1. (max = 12)	5. (max = 9)
2. (max = 15)	6. (max = 6)
3. (max = 12)	7. (max = 22)
4. (max = 8)	
Final Score _________(max=84)	

1) (4 pts each, 12 pts total) **2D Transforms**: What is the 3x3 matrix transform (or sequence of transforms) in homogeneous coordinates for the following. Also give the inverse.

a) A translation by 3 along x and -4 along y.

The transform:

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

The inverse:

$$\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

b) A uniform scale by 2 about the fixed point (a,b):

The transform:

$$\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix}$$

The inverse:

$$\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix}$$

c) A projection onto the y-axis.

The transform:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

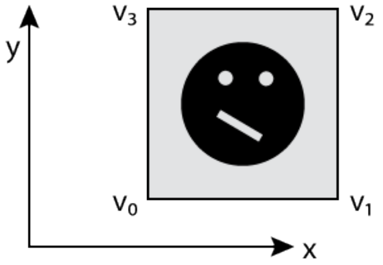
The inverse:

does not exist

2) (5 pts each, 15 pts total) Suppose you want to apply the following texture to the square surface on the right.

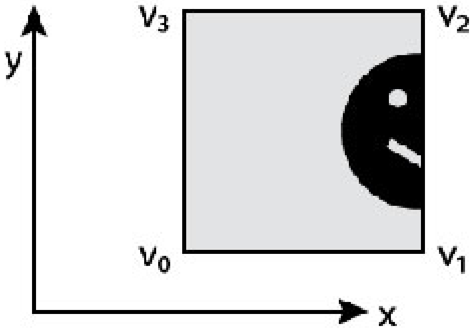
If the code below is applied, the square will be textured as shown below on the right.

```
gl.glTexParameteri(GL.GL_TEXTURE_2D,
   GL.GL_TEXTURE_WRAP_S.GL.GL_ CLAMP);
gl.glTexParameteri(GL.GL_TEXTURE_2D,
   GL.GL_TEXTURE_WRAP_T.GL.GL_CLAMP);
gl.glBegin(GL.GL_POLYGON);
  gl.glTexCoord3f(0,0);   gl.glVertex3f(v0x,v0y,0)
  gl.glTexCoord3f(1,0);   gl.glVertex3f(v1x,v1y,0);
  gl.glTexCoord3f(1,1);   gl.glVertex3f(v2x,v2y,0);
  gl.glTexCoord3f(0,1);    gl.glVertex3f(v3x,v3y,0);
gl.glEnd();
```

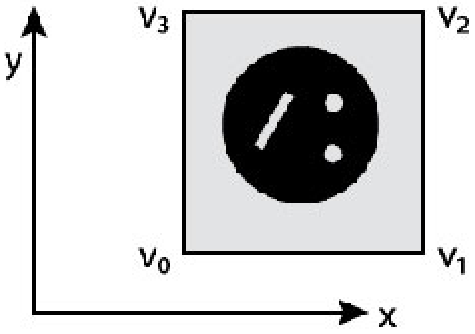


In the rectangle on the right, sketch how the texture would appear if the following texture coordinates are used instead?

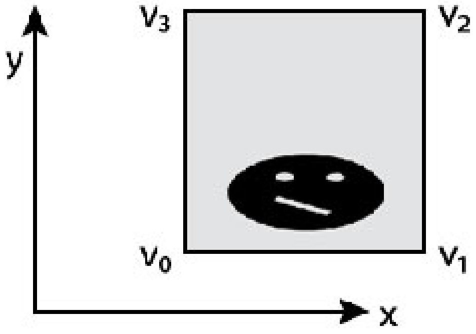
a.
```
gl.glTexCoord3f(-0.5,0);   gl.glVertex3f(v0x,v0y,0);
gl.glTexCoord3f(0.5,0);   gl.glVertex3f(v1x,v1y,0);
gl.glTexCoord3f(0.5,1);   gl.glVertex3f(v2x,v2y,0);
gl.glTexCoord3f(-0.5,1);    gl.glVertex3f(v3x,v3y,0);
```



b.
```
gl.glTexCoord3f(1,0);   gl.glVertex3f(v0x,v0y,0);
gl.glTexCoord3f(1,1);   gl.glVertex3f(v1x,v1y,0);
gl.glTexCoord3f(0,1);   gl.glVertex3f(v2x,v2y,0);
gl.glTexCoord3f(0,0);    gl.glVertex3f(v3x,v3y,0);
```

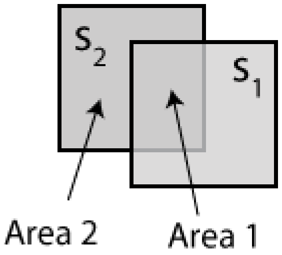


c.
```
gl.glTexCoord3f(0,0);   gl.glVertex3f(v0x,v0y,0);
gl.glTexCoord3f(1,0);   gl.glVertex3f(v1x,v1y,0);
gl.glTexCoord3f(1,2);   gl.glVertex3f(v2x,v2y,0);
gl.glTexCoord3f(0,2);    gl.glVertex3f(v3x,v3y,0);
```



3) (4pts each, 12 pts total) Blending in OpenGL: Suppose that you draw two rectangles S_1 and S_2, using OpenGL where

- S_1 is closer to the camera than S_2.
- S_1 is drawn first followed by S_2.
- The background color is set to white=(1,1,1).
- the glColor is set to green=$(0,1,0, \alpha_1)$ when S_1 is drawn.
- the glColor is set to red=$(1,0,0,\alpha_2)$ when S_2 is drawn.



What is the *resulting (r,g,b) color in the frame buffer* corresponding to each of the following. *Show how you calculated your answer* (you will not get full credit for just an answer written down)

a. **Area 2** if α_1=0.5, α_2=0.6 when z-buffering is turned on.

(r,g,b) = ___________

In area 2, S1 has no effect.
$(1-\alpha_2)$ (1,1,1) + (α_2)(1,0,0) = (.4)(1,1,1)+(.6)(1,0,0) = (1,.4,.4)

b. **Area 1** if α_1=0.4, α_2=1.0 when z-buffering is turned on.

(r,g,b) = ___________

S2 has no effect since z buffering is on and S1 is drawn first (and is in front of S2).
$(1-\alpha_1)$ (1,1,1) + (α_1)(0,1,0) = (.6)(1,1,1)+(.4)(0,1,0) = (.6,1,.6)

c. **Area 1** if α_1=0.25, α_2=0.9 when z-buffering is turned off.
(r,g,b) = ___________
Both S1 and S2 have an effect since they overlap in area 1 and z-buffering is off
When S1 is drawn:
$(1-\alpha_1)$ (1,1,1) + (α_1)(1,0,0) = (.75)(1,1,1)+(.25)(0,1,0) = (.75,1., .75)
Then, when S2 is drawn:
$(1-\alpha_2)$ (.75,1., .75) + (α_2)(1,0,0) = (.1)(. 75,1., .75)+(.9)(1,0,0) = (.975, .1, .075)

(4 pts each, 8 pts total) **Fly thoughs**: Given the viewer location *Loc*, and the direction/orientation vectors *Forward*, *Right*, and *Up*, how do you update *Loc, Forward, Right, and Up* in order to do the following. Indicate not only what changes but also what doesn't change.

a) Move the viewer a small distance (e.g. α) forward.

   Loc = Loc + α * Forward
   Forward, Right, Up do not change

b) Turn the viewer a small amount to the left.

   Loc and Up do not change
   Forward = (Forward – α * Right) / || Forward – α * Right ||
   Right = (Forward X Up) / || Forward X Up ||

4) (3 pts each, 9 pts total) The position of a light can be set in any number of places in your code, e.g. before or after gluLookAt, at some point in scene hierarchy, etc. Where should the position be set if

a) You want the light to move with the camera?

   Place it before gluLookat

b) You want the light to stay fixed in the world coordinate system?

   Place it after gluLookAt

c) You want the light to be attached to an object (e.g. if you had a headlight on a car).

   Place it in the code immediately before drawing the object.

5) (6 pts) Cross Products: If v1 = (1,0,0) and v2 = (0, 0, 1), what is

a) v1 x v2 = ____(0, -1, 0) __________

b) v1 x v1 = ______(0,0,0) ________

6) (22 pts total) Recall that *OpenGL uses a right handed coordinate system where the default camera looks down the negative z world axis, the y axis is up, and the x axis is to the right.* In this default position, the world coordinates of a vertex p_w are the same as its camera coordinates p_e.

a) (3 pts) Suppose the camera starts in the default position and is translated one unit back (along positive z). If a vertex has world coordinates p_w = (0,0,0), what would be its new camera coordinates, p_e? (no math required – just reason it through)

(0, 0 , -1)

b) (3 pts) Suppose the camera starts in the default position and is rotated by 90 degrees about x so that it is looking directly up along the y world axis. If a vertex has coordinates p_w = (0,0,-1), what would p_e be? (no math required – just reason it through)

(0, -1, 0)

(Continued on next page)

c) (16 pts) Suppose we want to reposition the camera given the parameters:

Eye = position of the camera
Look = what the camera is looking at
Up = the up vector

We can use the function gluLookAt(*Eye*, Look, Up) to calculate M, where M is the 4x4 matrix used to transform vertices p_w (world coordinates) into p_e (eye coordinates):

$$p_e = M\, p_w$$

How does gluLookAt calculate M from Eye, Look, and Up? That is, what is M in terms of Eye, Look, and Up? (Hint: first calculate camera coordinate vectors u,v,n and use them to compute the rotational part of M. Then determine the translational part of M and combine with the rotational part to get to M. Use homogeneous coordinates.)

$n =$ (Eye – Look) / || (Eye – Look) ||
$u =$ (Up x n) / || (Up x n) ||
$v =$ n x u

$$M_{rot} = \begin{pmatrix} ux & ux & ux & 0 \\ vy & vy & vy & 0 \\ nz & nz & nz & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad M_{trans} = \begin{pmatrix} 1 & 0 & 0 & -eyex \\ 0 & 1 & 0 & -eyey \\ 0 & 0 & 1 & -eyez \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$M = M_{rot}\, M_{trans}$$