



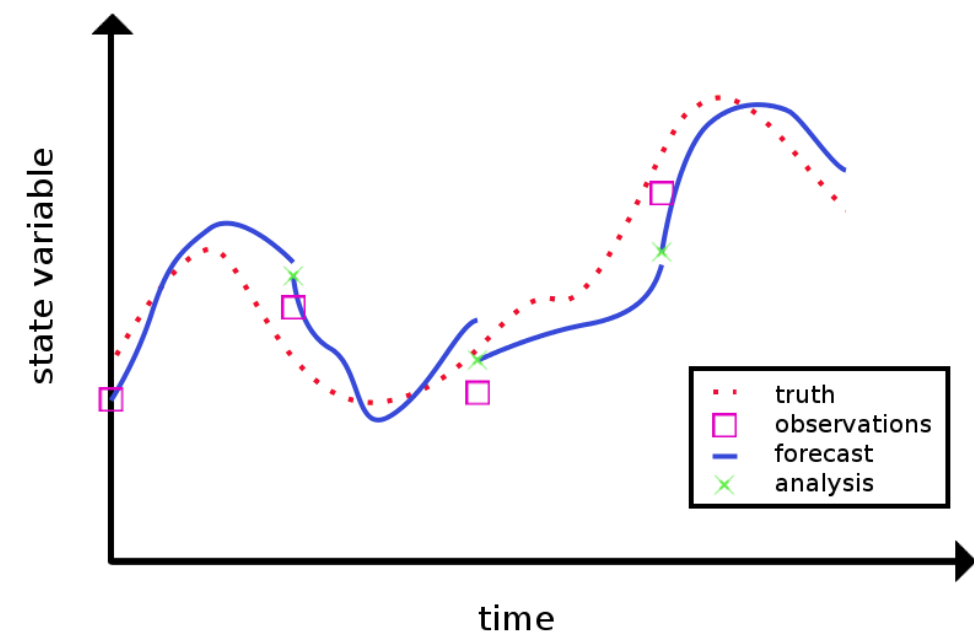
Adaptive Data Assimilation Scheme for Shallow Water Simulation

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Problem and Motivation



- Current data assimilation (DA) combines observation data with model forecast to produce more accurate prediction
- Assimilation occurs **sequentially** and each observation carries equal importance despite **unequal improvement to accuracy**
- Because of dramatic increase in available observation data, assimilating whenever observation becomes available is **impractical**

Proposed Improvements

- Use Kullback-Leibler distance to measure **information gain** from an observation
- Use this **metric** as criteria to **control DA frequency** to make EnKF adaptive.

Kullback-Leibler Distance, and Ensemble Kalman Filter

- The Kullback Leibler distance (KLD) measures the similarity between **probability distribution functions (PDFs)**
- Non-semetric measurement between two PDF's: P, Q

$$D_{KL}(P||Q) = \int p(x) \ln(p(x)/q(x)) dx \quad (1)$$

- In our calculations we used the discretized version of KLD.

$$D_{KL}(P||Q) = \sum P(i) \ln(P(i)/Q(i)) \quad (2)$$

- The **Ensemble Kalman Filter** is a recursive filter commonly used in geophysics models
- The filter consists of **three phases**:

(1) Compute Kalman gain

$$K_k = P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} \quad (3)$$

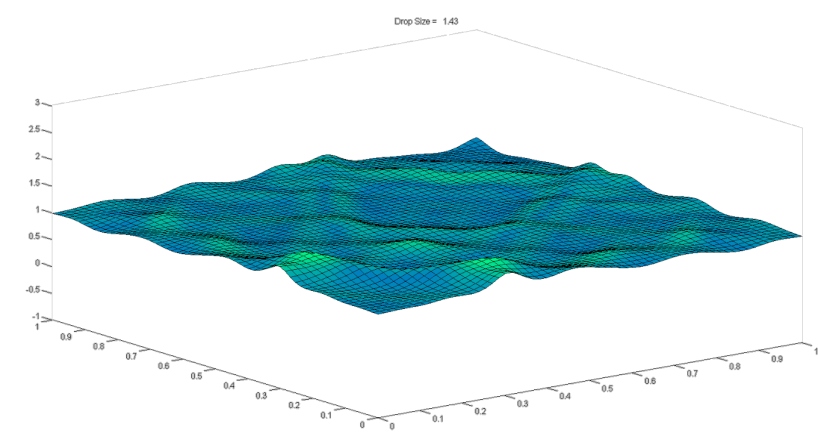
(2) Update estimate with measurement z_k

$$x_k = x_k + K(z_k^- - H_k x_k^-) \quad (4)$$

(3) Update the error covariance

$$P_k = (I - K_k H_k) P_k^- \quad (5)$$

Shallow Water Simulation



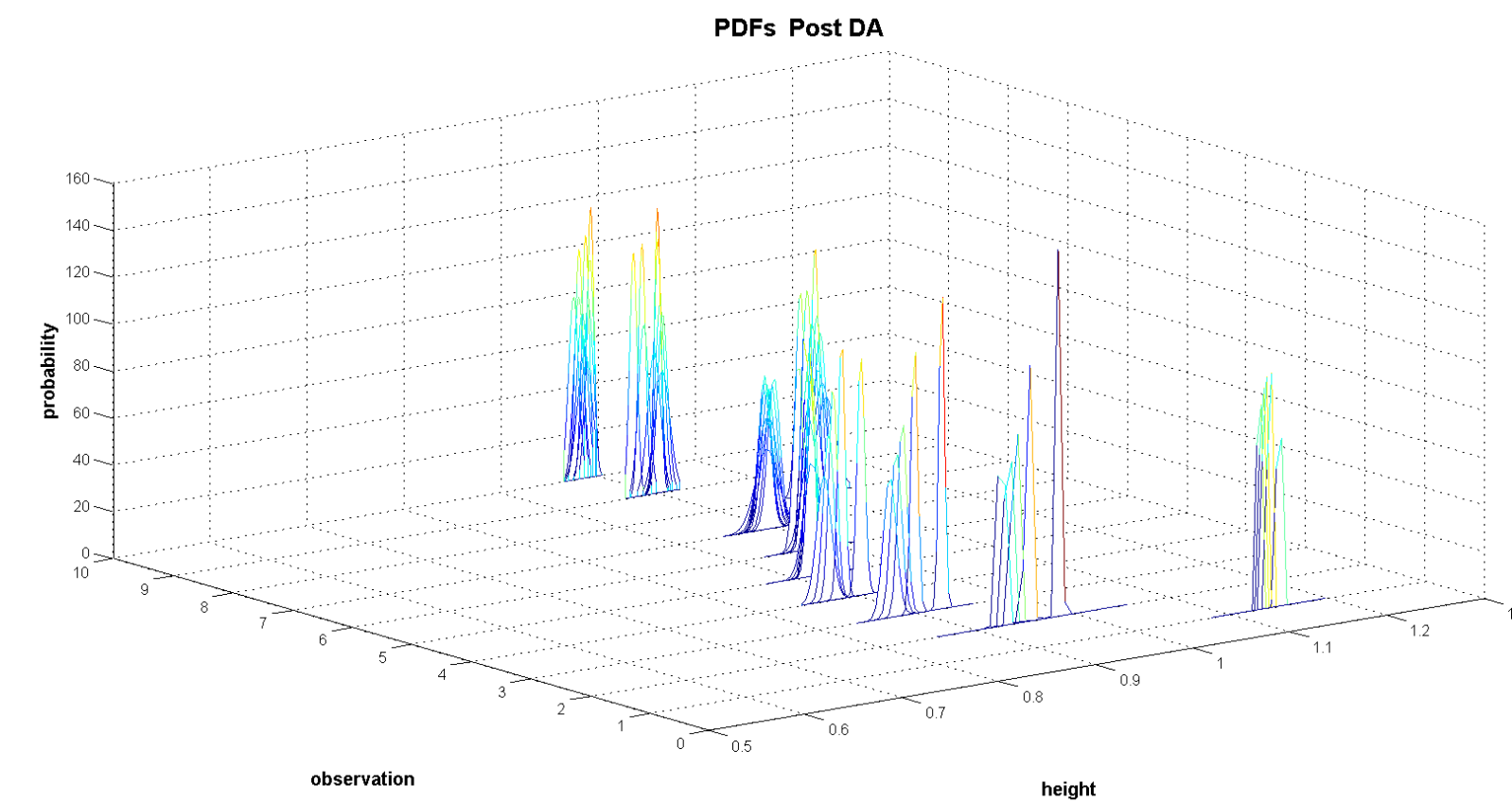
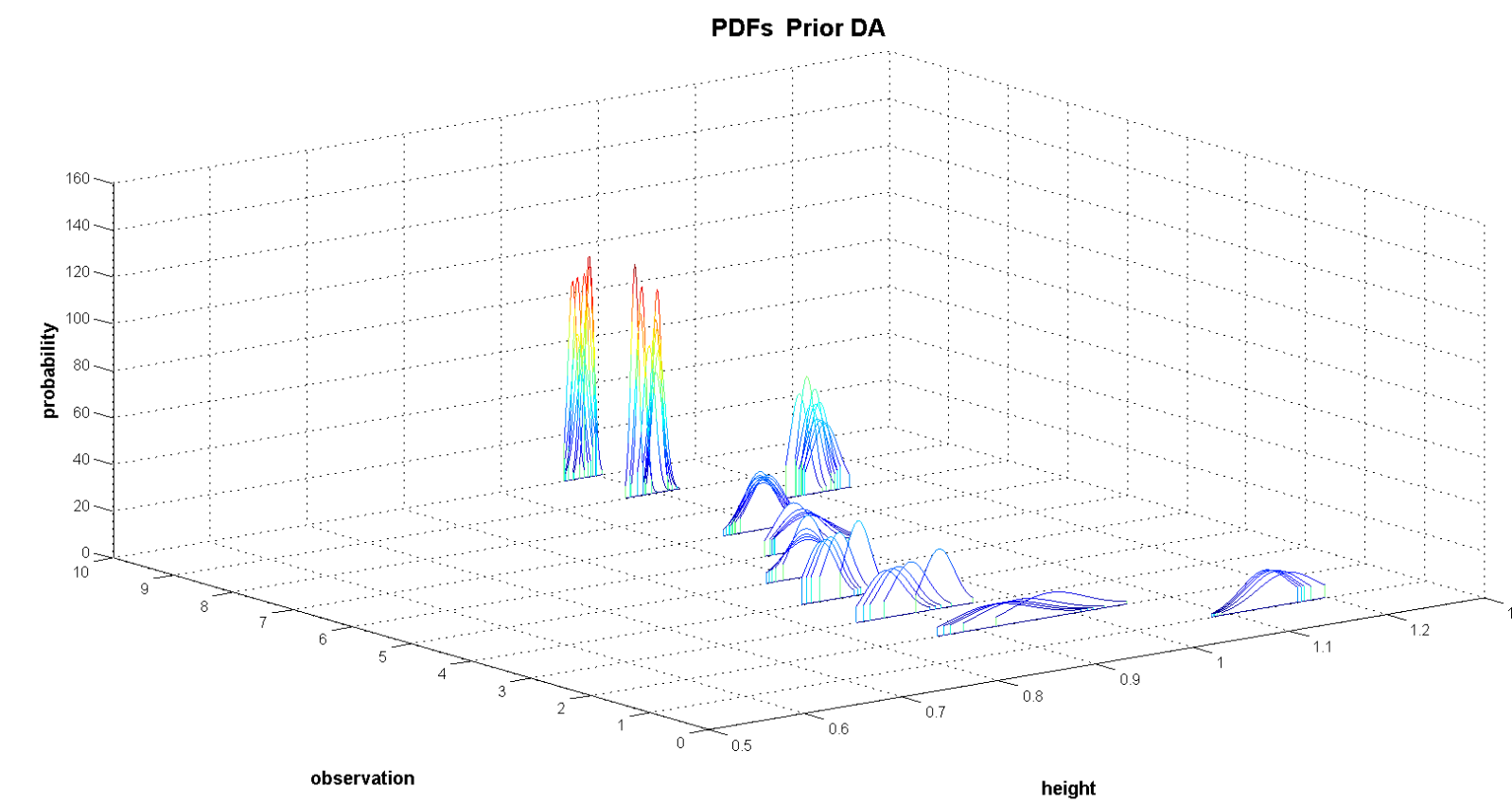
- Our model simulated **245760** state variables for **1000** steps.
- Grid: **64x64**, Ensemble Size: **20**, Number of State Variables: **3** (Height **H**, Velocity **U**, Velocity **V**)
- The model uses **three equations** from physics

$$\frac{\mu H}{\mu t} + \frac{\mu(HU)}{\mu x} + \frac{\mu(HV)}{\mu y} = 0 \quad (6)$$

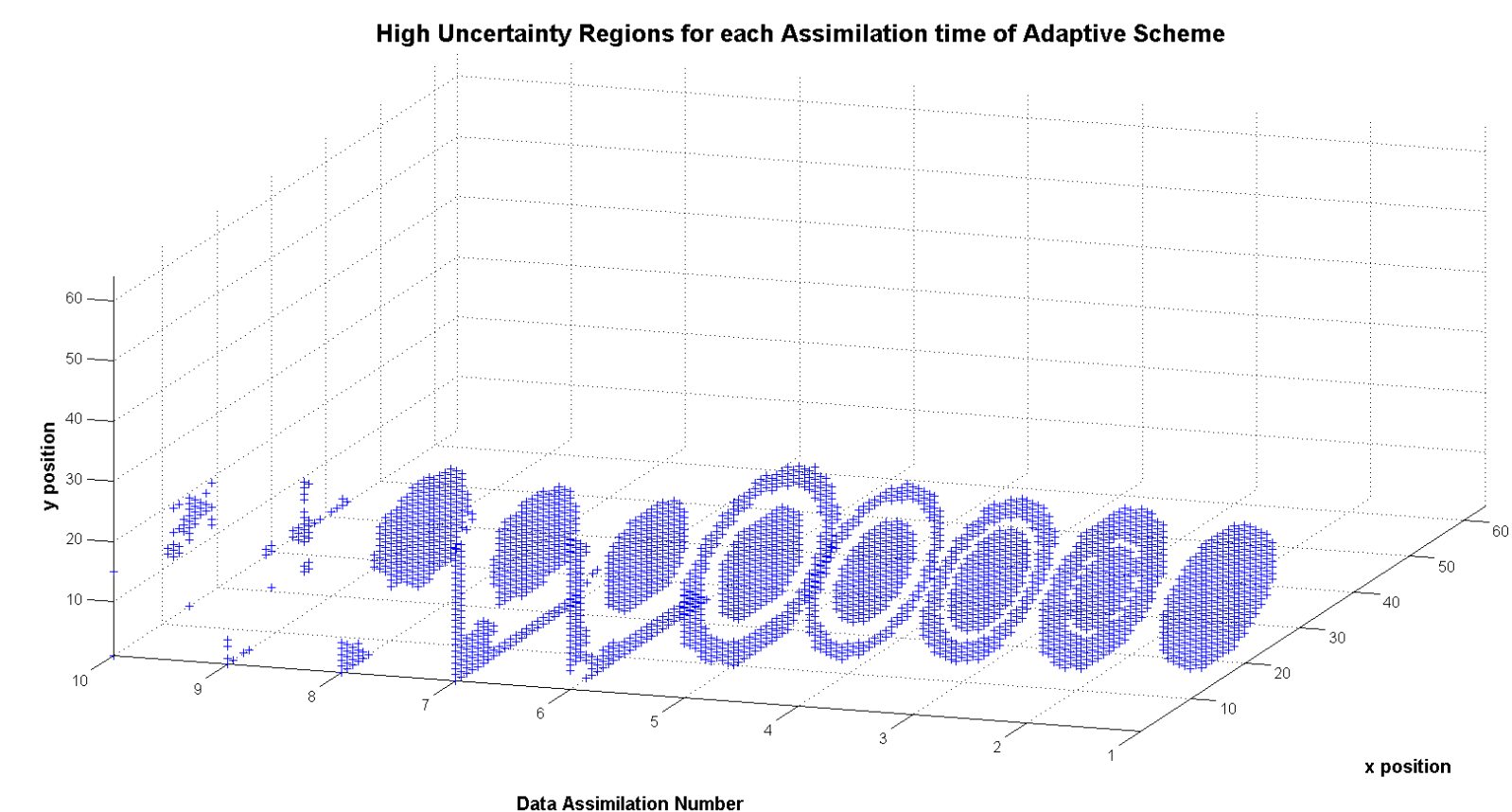
$$\frac{\mu HU}{\mu t} + \frac{\mu}{\mu x} (HU^2 + \frac{1}{2} g H^2) + \frac{\mu(HUV)}{\mu y} = 0 \quad (7)$$

$$\frac{\mu HV}{\mu t} + \frac{\mu(HUV)}{\mu x} + \frac{\mu}{\mu y} (HV^2 + \frac{1}{2} g H^2) = 0 \quad (8)$$

Numerical Experiments

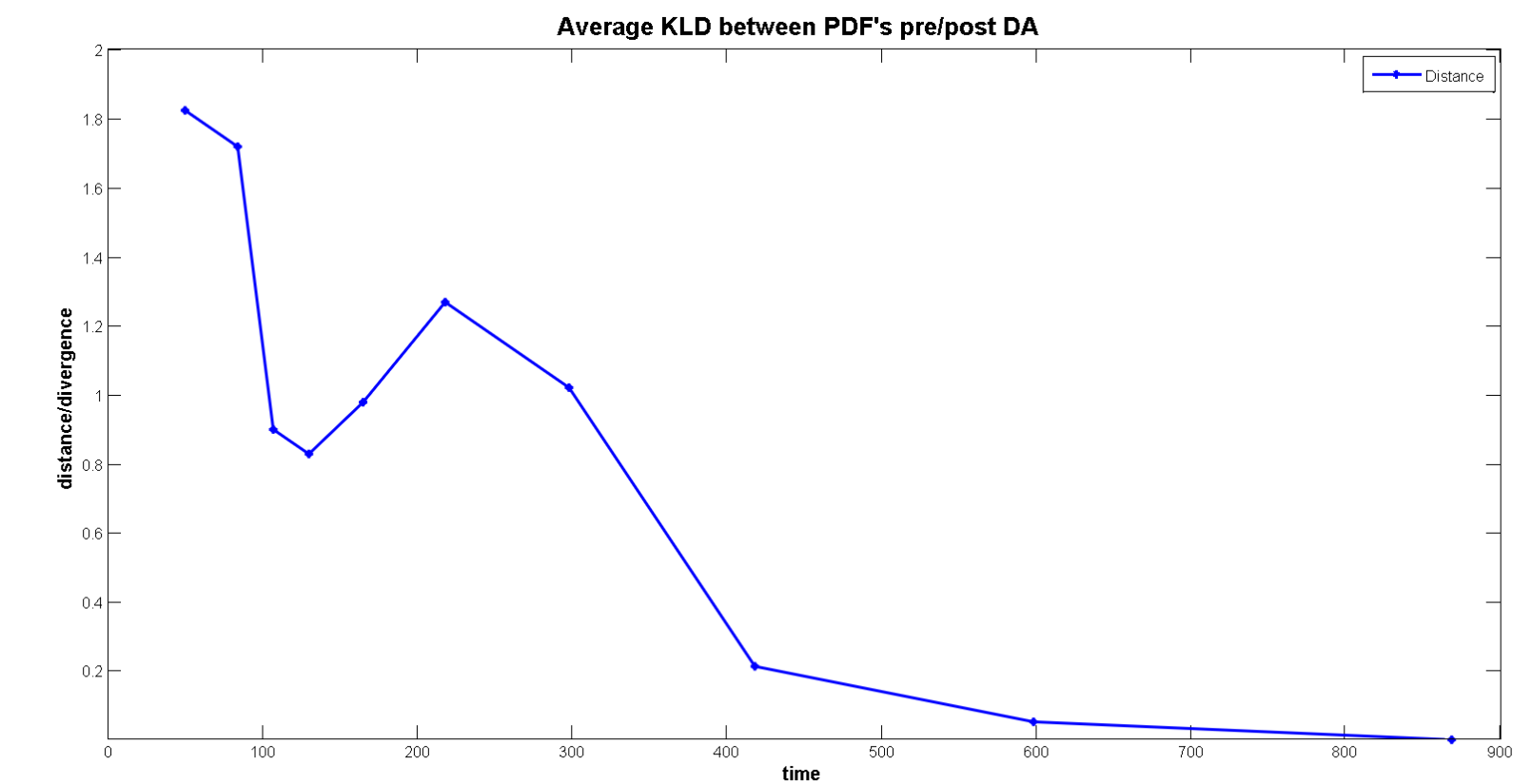


- PDFs built from 20 member implementation of **Ensemble Kalman Filter**, a common filter method used in geophysics simulations.
- Width** of PDFs represents **deviation** of ensemble
- Height** of PDFs represents **likelihood** of values

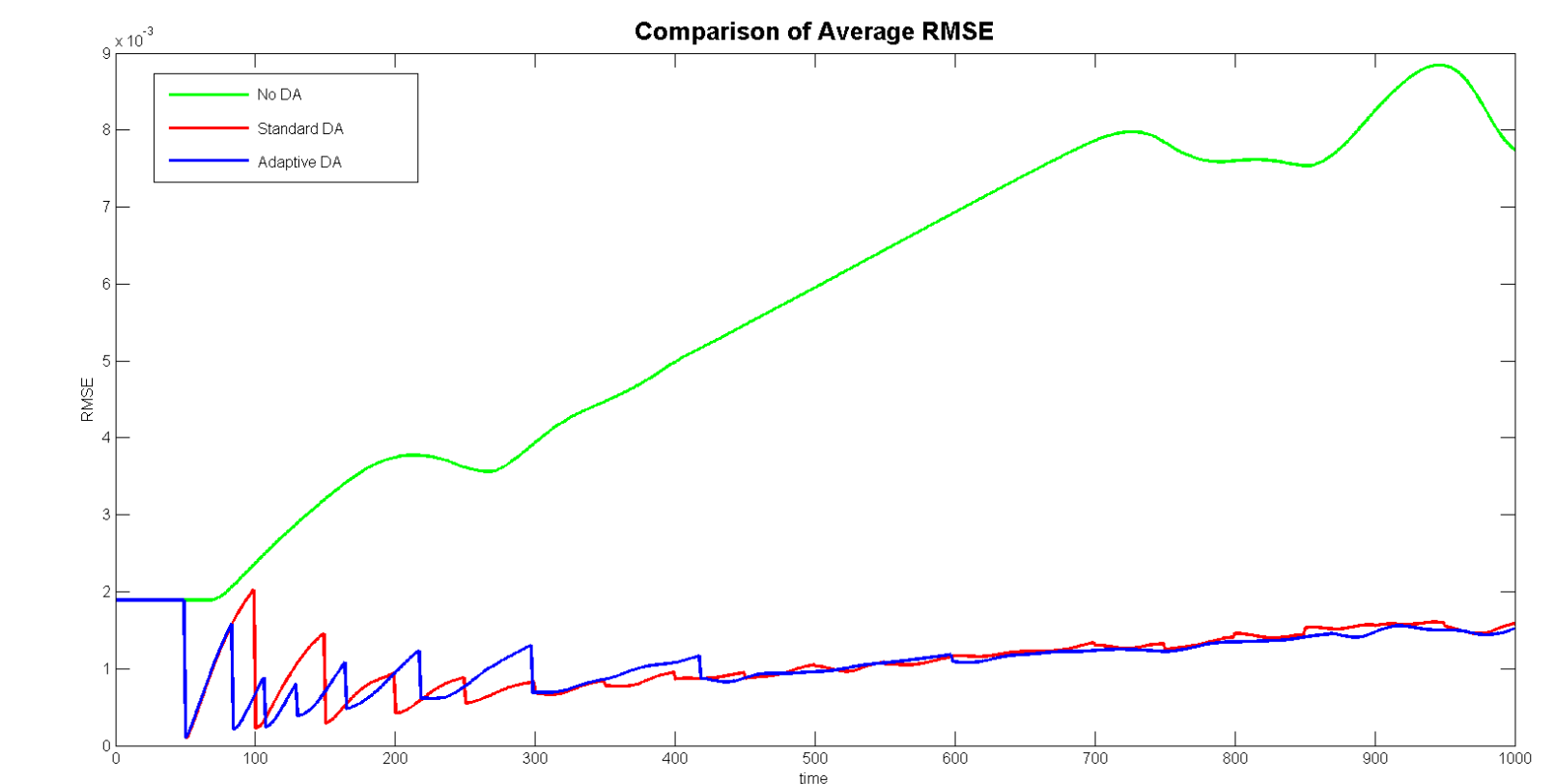


- Locations in results with greatest uncertainty (**PDFs max height greater than 100**)
- The **disappearance** over time of **high uncertainty areas** show that the assimilation scheme is effective

Results



- After several assimilations, the KLD dropped significantly, and correspondingly, there was minimal gain from more assimilations



- Average Root Mean Square error comparison for three versions of shallow water model: No DA, Standard EnKF DA, and Adaptive EnKF DA.

Conclusion

Standard(secs)	Adaptive(secs)	Number of Observations:	
		Standard	Adaptive
822.1663	663.7627	20	11
812.9363	653.7945	20	10
812.3552	592.3358	20	9
905.6271	718.1229	20	10
862.8993	609.4673	20	9
930.4412	676.9176	20	10

- Numerical results show the adaptive EnKF scheme can achieve **similar numerical accuracy** with a lower runtime and **fewer assimilations**