

Uncertainty Quantification with a Non-Probabilistic Approach

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Abstract

The majority of past research regarding uncertainty quantification has been executed with a probabilistic approach. The combination of Fuzzy Logic and Dempster-Shafer theory presents a new frontier for uncertainty quantification with a non-probabilistic approach. Uncertainty Quantification reduces uncertainty and helps make decisions in many fields of research. In this work we designed a tool to simulate and test an existing framework used in an object identification application. The software provides a flexible user interface to facilitate testing different scenarios.

Introduction

Uncertainty Quantification:

Uncertainty quantification is a process of the quantitative characterization and reduction of uncertainties in applications. Most research in UQ (uncertainty quantification) and UR (uncertainty reduction) have been implemented by probabilistic approaches, mainly with the Monte Carlo method. When using Monte Carlo Method, the optimum result depends on a large number of realizations. Which is not applicable for large models. Thus non-probability approaches provide an alternative to quantify uncertainty. There are three non-probabilistic approaches: Fuzzy Logic, Dempster-Shafer Theory, and Interval Analysis. Each has advantages and disadvantages. One framework was proposed to combine Fuzzy Logic and Dempster-Shafer Theory ([1], [2], [3], [4], [5], [6]), where the Fuzzy Logic is used to represent membership functions and the Dempster-Shafer theory is used to combine information coming from different independent sources. This research involves developing an automated test software to assess the effectiveness of this framework.

Object Identification Problem:

The following example explains the problem of object identification: Assume there are three objects, each with two features: size and color. Let the color take on two values red and blue. Let the size take on two values large and small.

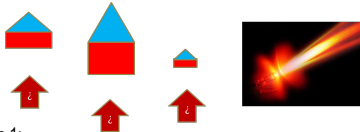


Figure 1:
Assume there exists a sensor that reports values of certain features but in an imperfect manner. Assume this sensor is pointing to one of the objects. But which one? The goal is to draw a reasonable conclusion as to what object is the focus of our sensor. This is the problem of object identification.

Fuzzy Logic and Evidence Theory (Dempster-Shafer)

Fuzzy Logic is a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information.

Fuzzy Logic incorporates a simple rule, If X AND Y THEN Z approach to solving control problems. e.g. If (water is cold) AND (water is getting colder) THEN (add heat to water).

Fuzzy Logic membership functions map values in the range between 0 and 1.

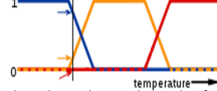


Figure 2:

Pros:

- 1) It's robust, so it doesn't require precise, noise free inputs
- 2) It can be programmed to fail safely

Cons:

- 1) System complexity increases rapidly with more inputs and outputs since rules defining their interrelations must also be defined

Dempster-Shafer Theory is a mathematical theory of evidence. It allows one to combine evidence from different sources and arrive at a degree of belief that takes into account all the available evidence.

There are two critical and related issues concerning the combination of evidence from multiple sources:

- 1) The type of evidence involved and 2) How to handle conflicting evidence

Types of Evidence:

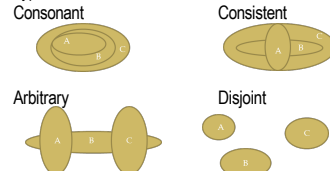


Figure 3:

Dempster-Shafer Theory emphasizes the agreement between multiple sources and ignores all the conflicting evidence through a normalization factor. This can sometimes lead to a false conclusion. For example: Suppose one Doctor believes a patient has meningitis with .99% or a brain tumor with .01%. Suppose another Doctor believes the same patient has a concussion with .99% or a brain tumor with .01%. Then using Dempster-Shafer Theory the patient has a brain tumor, which is an incorrect result.

Framework to Combine FL and D-S theory

The following equations are used to combine the Dempster-Shafer part with the Fuzzy Logic part:

Figure 4a:
$$c_{ja} = \sum_t s_{jt} \min\{u_{it}^{k_j}\}$$

The maximum payoff is calculated by c_{ja} here $u_{it}^{k_j}$ is the membership of objects in fuzzy set F and s_{jt} a measure of how successfully object j may replace object t.

The object is chosen by maximizing the average payoff using the following equation:

Figure 4b:
$$\max_j \sum_a c_{ja} m(S_a)$$

Where c_{ja} the maximum payoff and s_{jt} notes the probability of the environment

Software and Simulation Test

The research developed a program to simulate and test an existing framework on object identification. It allows one to automate a test case. Here is a screen shot of the program menu:

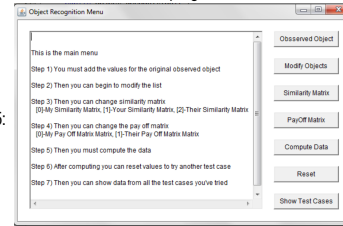


Figure 5:

From this menu, it is possible to do the following: set the values of the observed object, modify the values of the objects, modify the similarity matrix, and choose between their payoff equation or the one I came up with. The compute data button calculates what object was being observed by the sensor. The reset button allows you to reset all or only certain values back to zero, so you can try a different test case. The show test cases button lists all the data from all the test cases you've tried since running the program

In this specific test case, there were three objects with the following values (denoted by u_{it} , where t is the row and i is the column) shown on figure 6 below:

Features	Object 1	Object 2	Object 3
Blue	0.8	0.6	0.3
Red	0.5	0.7	0.8
Large	0.6	0.9	0.5
Small	0.4	0.7	0.6

The observed object had the following values shown on figure 7:

Features	Observed Object
Blue	0.7
Red	0.3
Large	0.8
Small	0.2

The similarity matrix had the following values (denoted by s_{jt} shown on figure 8 below:

	1	2	3
1	1.0	0.7	0.6
2	0.7	1.0	0.8
3	0.6	0.8	1.0

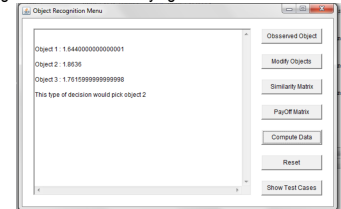
The environment (denoted by S_a) is defined by taking the outer product of the color and size features of the observed object. This is done by multiplying blue with large, blue with small, red with large, and red with small. Which gives us E1 = blue * large = 0.7 * 0.8 = 0.56, E2 = blue * small = 0.14, E3 = red * large = 0.24, E4 = red * small = 0.06.

After all this information is given, the equations in figure 4 are used to compute the results. This is shown by figure 9 below:

Objects	1	2	3
Results	1.211	2.716	1.21456

In the end, the sensor in the test case was reporting values from object 2. Plugging the given values from the test case into the developed program will produce the exact same results as above.

This specific object identification problem only provided one test case. Using the developed software, I decided to try an extreme case by making all the values from the sensor match all the values from the third object in our list. Using all their settings, I ran the program. Results shown by figure 10 below:



The authors' process chose object 2 as the object the sensor was looking at. Even though, I made the sensor have all the values from object three. In the end, their process does not accurately solve the object identification problem. More research needs to be done to find a better working algorithm.

Summary

Research initialized research with uncertainty quantification with a non-probabilistic approach. A program was developed in this research to automate the test process of a framework that combines Fuzzy Logic and Dempster-Shafer Theory to quantify and reduce uncertainty. The test result revealed some problems with the proposed framework. A better strategy needs to be designed to improve the framework.

References

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