



# Numerical Analysis of the Hybrid Particle Filter on a Chaotic System



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## OBJECTIVES

Our goal is to reduce the effects of the infamous problem of particle degeneration in Particle Filtering (PF). We attempted to tackle it by hybridizing the PF with an additional algorithm called Particle Swarm Optimization (PSO). We apply the hybridized PF to the Lorenz-63 and Lorenz-96 chaotic systems.

## PARTICLE FILTER

Particle Filtering (PF), also known as Sequential Monte Carlo (SMC), is a numerical method for improving the accuracy of state forecasts. It uses an ensemble of state samples – so-called “particles” – that can change over time. Each implementation of PF has a transition function that is responsible for either simulating or otherwise transitioning the states of the samples. PF’s ability to handle non-linear transitions, and non-Gaussian errors, gives it the advantage over other filtering methods.

State transition from time-step  $k - 1$  to  $k$ :

$$x_k^i = F_k(x_{k-1}^i, v_k)$$

Mapping the sample  $x_k^i$  to observation space:

$$y_k = H_k(x_k^i) + w_k$$

, where  $y_k$  is the observation at time-step  $k$ . The weights are updated according to:

$$w_k^i = \frac{p(y_k|x_k^i) p(x_k^i)}{\sum_j^N p(y_k|x_k^j) p(x_k^j)}$$

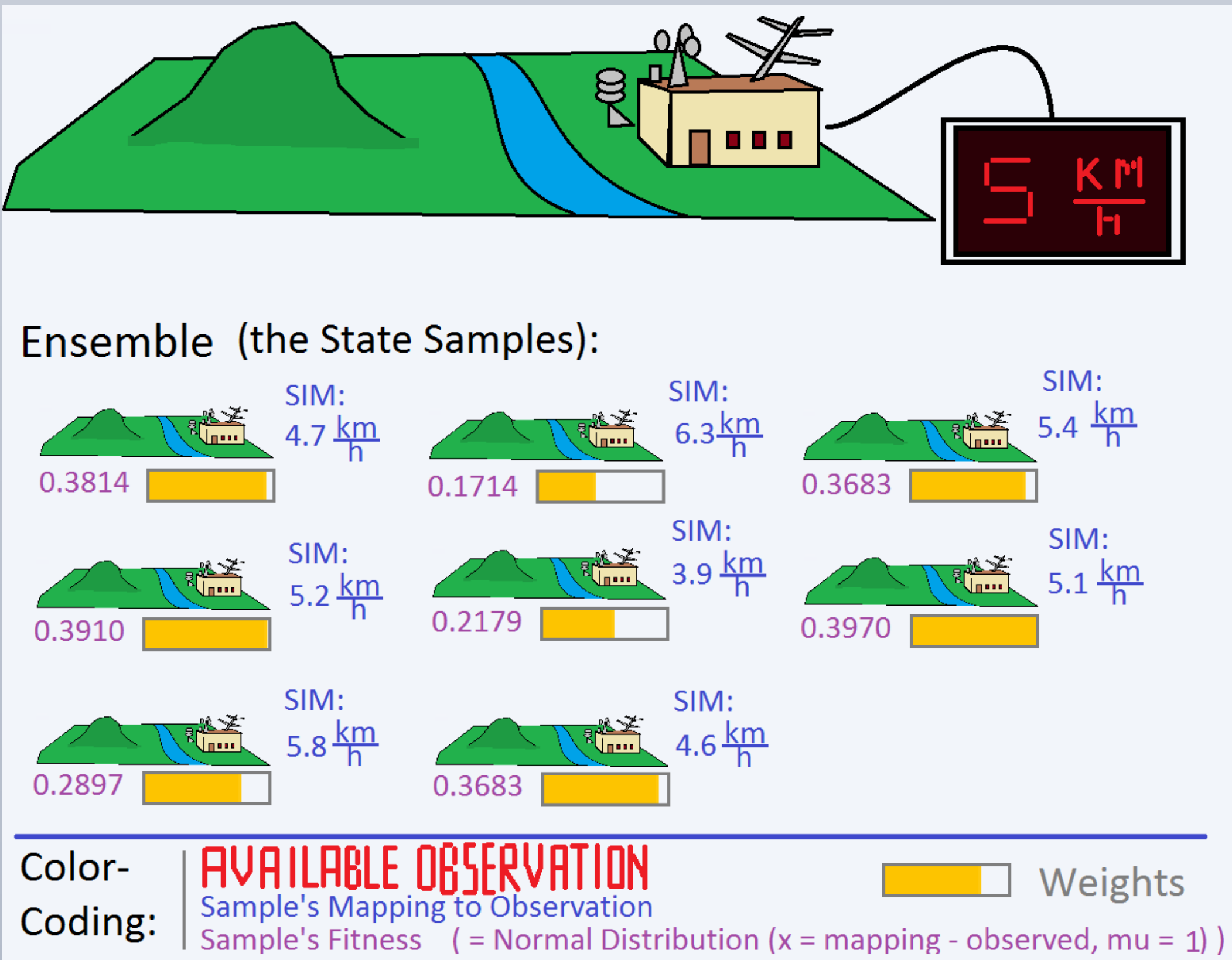
, where  $p(y_k|x_k^i)$  is the probability of the observation given the state of the  $i$ -th sample, and  $p(x_k^j)$  is the previous weight of the particle.

The *effective sample size*, defined by:

$$N_{eff} = \frac{1}{\sum_{i=0}^N (w_k^i)^2}$$

is an indicator of particle degeneration.

## EXAMPLE PF – WEATHER APPLICATION



## PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) takes advantage of the swarm nature of an ensemble of particles.

The velocities are determined by three parameters: inertia weight ( $iw$ ), cognitive weight ( $c_1$ ), and social weight ( $c_2$ ); by randomness, drawn from the uniform distribution between 0 and 1:  $r_1, r_2$ ; and by values determined on-the-fly. The states are updated according to:

$$x_m^i = x_{m-1}^i + v_m^i$$

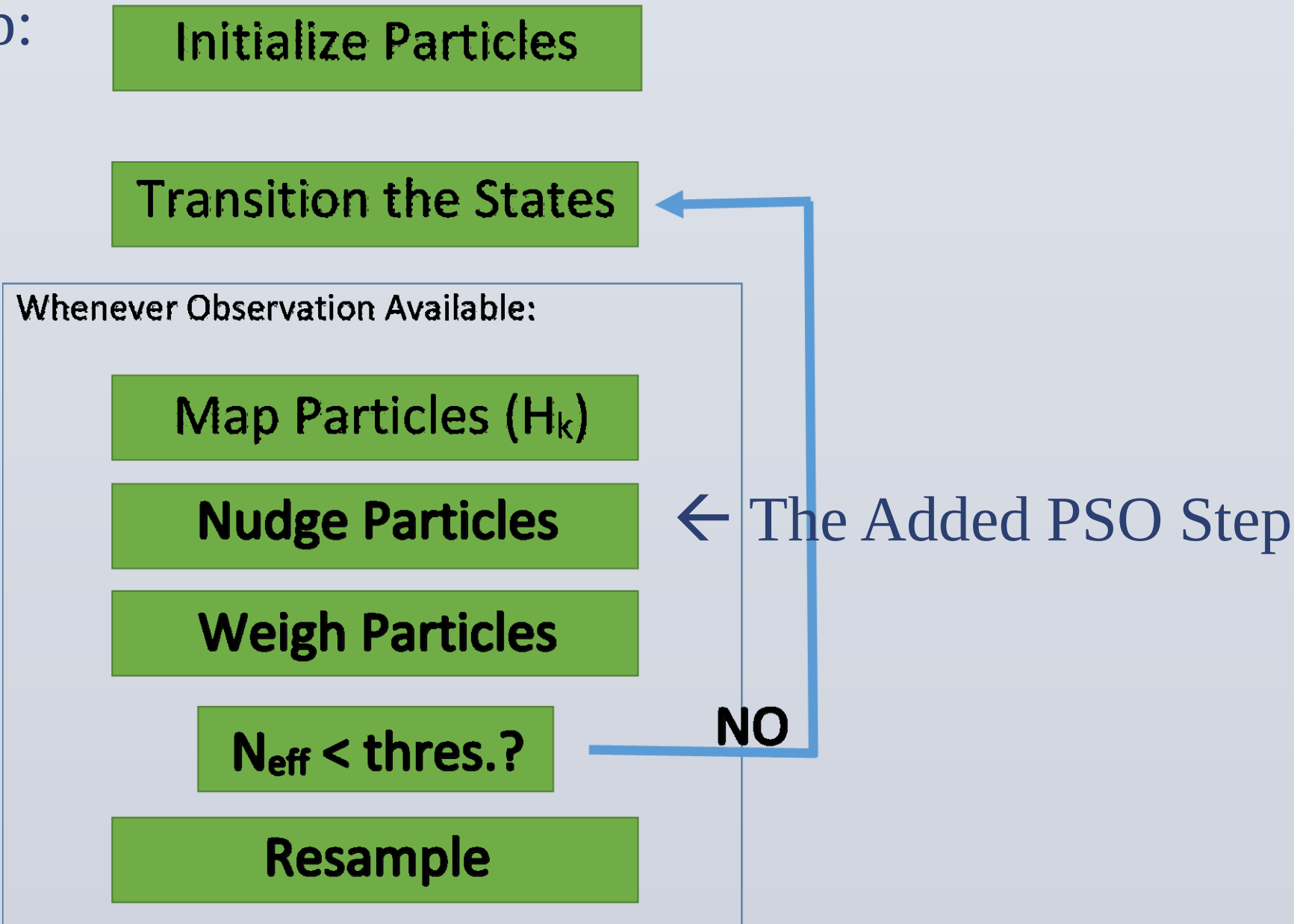
, in which

$$v_m^i = iw * v_{m-1}^i + r_1 * c_1 * (p_m^i - x_m^i) + r_2 * c_2 * (g_m - x_m^i)$$

$v_m^i$  is the velocity of the  $i$ -th particle in the  $m$ -th PSO iteration (PSO iterations are different from the PF time-step, and there may be many PSO iterations per time-step).

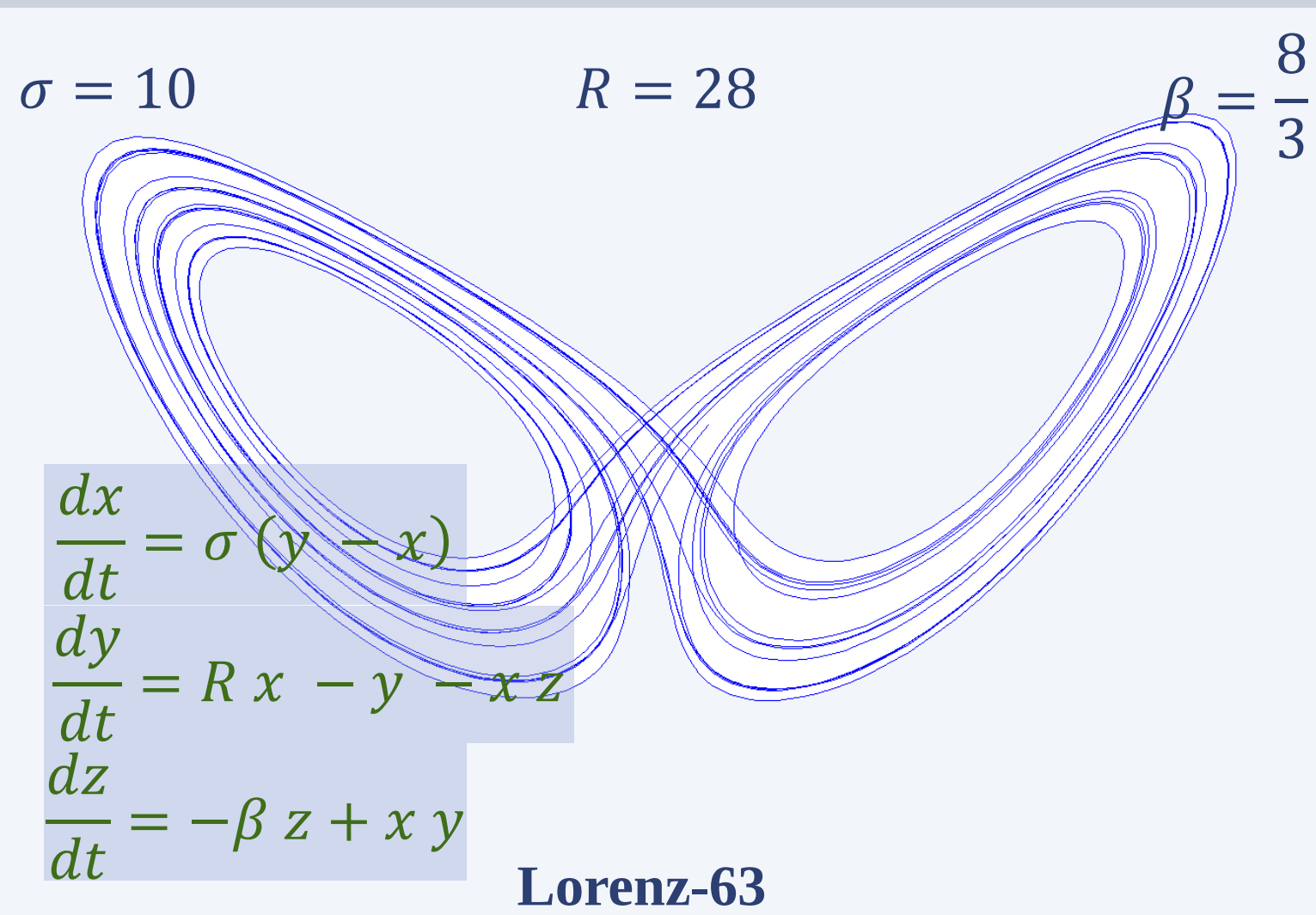
## PF-PSO HYBRID ALGORITHM

The hybridized PF-PSO algorithm works the same as classic PF except it has an added PSO nudging step:



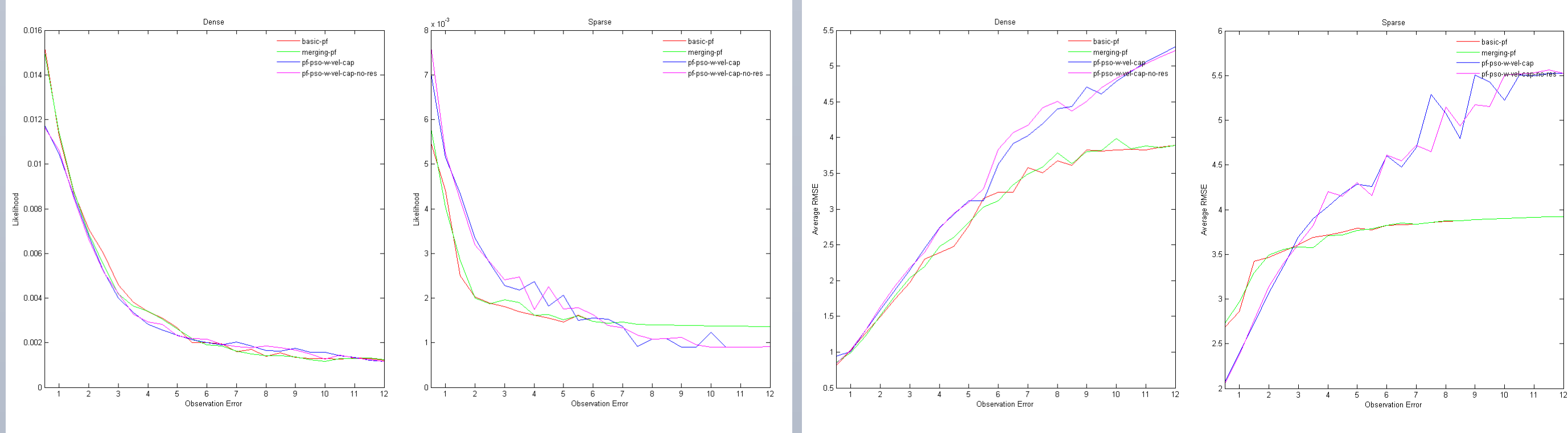
## NUMERICAL TESTS

(Right): The Lorenz-63 “butterfly” model we tested the filters on.



We also tested on the higher-dimensional Lorenz-96, which is not so easy to plot, but is still important.

The hybridized PF-PSO does work on the 3-dimensional Lorenz-63 model for low observational error:

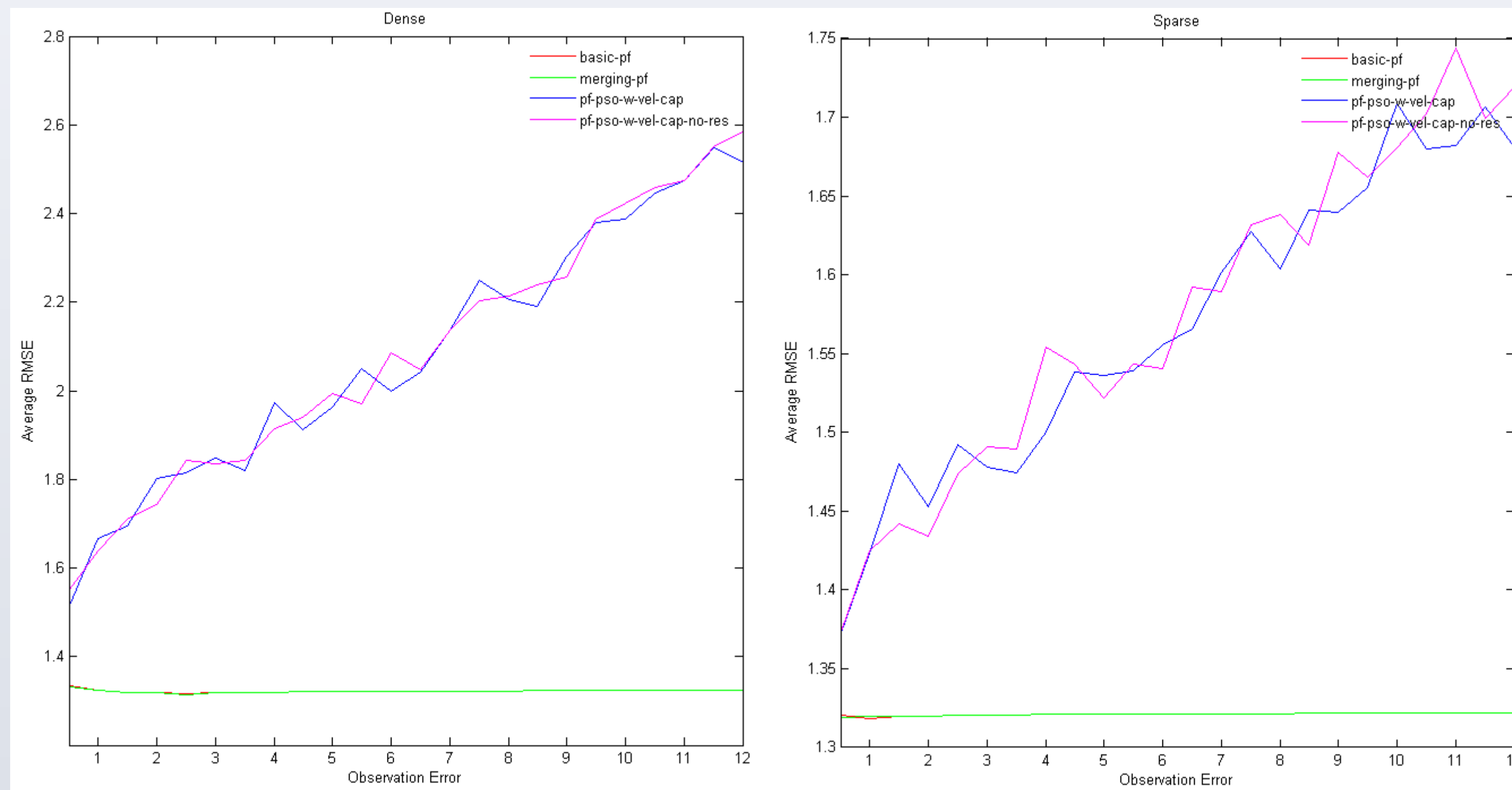


The likelihood of sampling the truth.

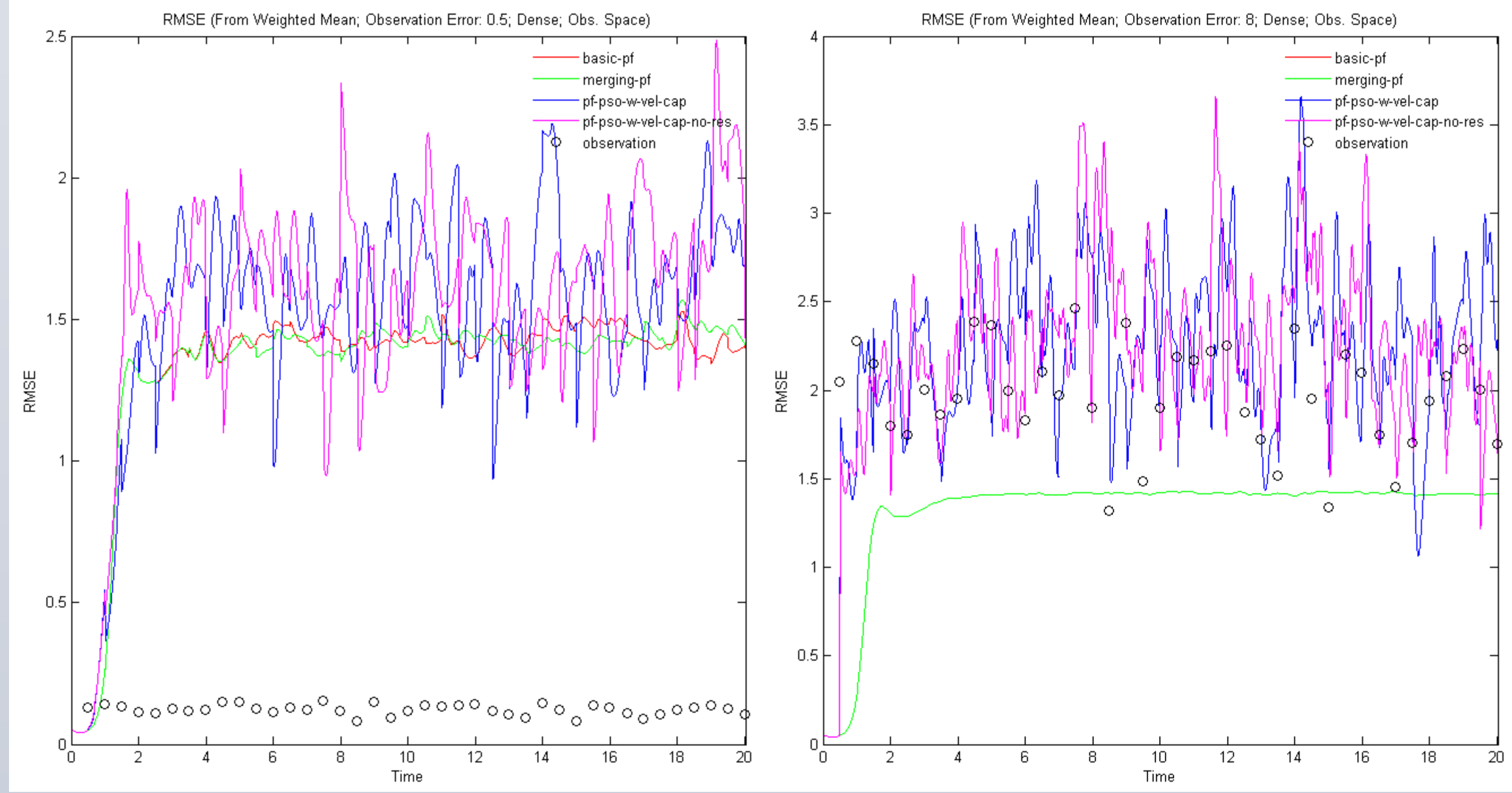
The RMSE from the same test.

## Lorenz-96

However, that is not the case on the Lorenz-96 chaotic model, which is 40-dimensional. To reduce the error, we changed the number of dimensions to 16 for our tests.



The following is a plot of how the filtering schemes behave over time, for two of the runs:



## CONCLUSIONS & FUTURE WORK

We applied the hybridized PF-PSO to two of the Lorenz chaotic systems, and it turned out that the additional PSO, although improves accuracy for the low-dimensional Lorenz-63, makes the filter perform worse for high dimensionality. It is probably because PSO drags states towards observations, improving some of the state components, but, meanwhile, other components are dragged away.

For future work, a breakthrough in the PF method itself is needed in order to make PF feasible for high-dimensional applications like weather forecasting.

## REFERENCES

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