MATH 141

Final Exam, Version 1

May 9, 2009

NAME (please print legibly): ____________________

Your University ID Number: ____________________

- **No calculators are allowed on this exam.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Answers such as** $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ **are perfectly fine!! However you MUST simplify expressions such as** $\sin(\pi/3)$**.**

useful formulae:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\overbrace{1 + 1 + 1 + \cdots + 1}^{\text{n times}} = n$$

Part A		
QUESTION	VALUE	SCORE
1	25	
2	10	
3	10	
4	27	
5	15	
6	13	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
1	15	
2	18	
3	18	
4	16	
5	15	
6	18	
TOTAL	100	

Part A

1. (25 pts) Limits. Calculate the following limits. Write DNE if the limit does not exist and is not $\pm\infty$.

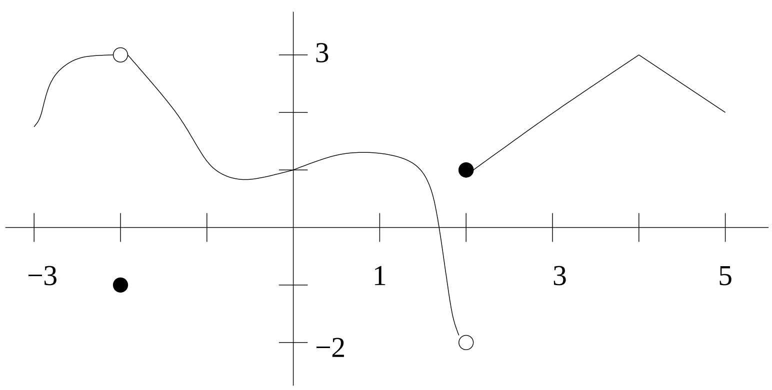
(a) $\lim_{x \to 0} \frac{x + \frac{2}{x}}{\frac{1}{x}}$

(b) $\lim_{x \to -1} \frac{x - 2}{x^2 - x - 2}$

(c) $\lim_{x \to 2} \frac{x - 2}{x^2 - x - 2}$

(d) $\lim_{x \to 1^-} \frac{x - 1}{|x - 1|}$

(e) Find the following limits from the graph of $g(x)$ given below.



$\lim_{x \to 2^-} g(x) =$ __________ $\lim_{x \to 3} \frac{g(x) - g(3)}{x - 3} =$ __________

$\lim_{x \to -2} g(x) =$ __________ $\lim_{x \to 4} g(x) =$ __________

2. (10 pts) Derivative formulas: The Product Rule. Give a proof of the product rule in the space provided below. All of the steps in the proof except the last step are included below, but these steps are OUT OF ORDER. Rewrite them in the correct order. Each step should follow easily from the next with no more than one algebraic manipulation.

Suppose f and g are differentiable functions.

$(f(x) \cdot g(x))' =$

OUT OF ORDER PROOF

$$= \lim_{h\to 0} f(x+h)\frac{(g(x+h)-g(x))}{h} + g(x)\frac{(f(x+h)-f(x))}{h}$$

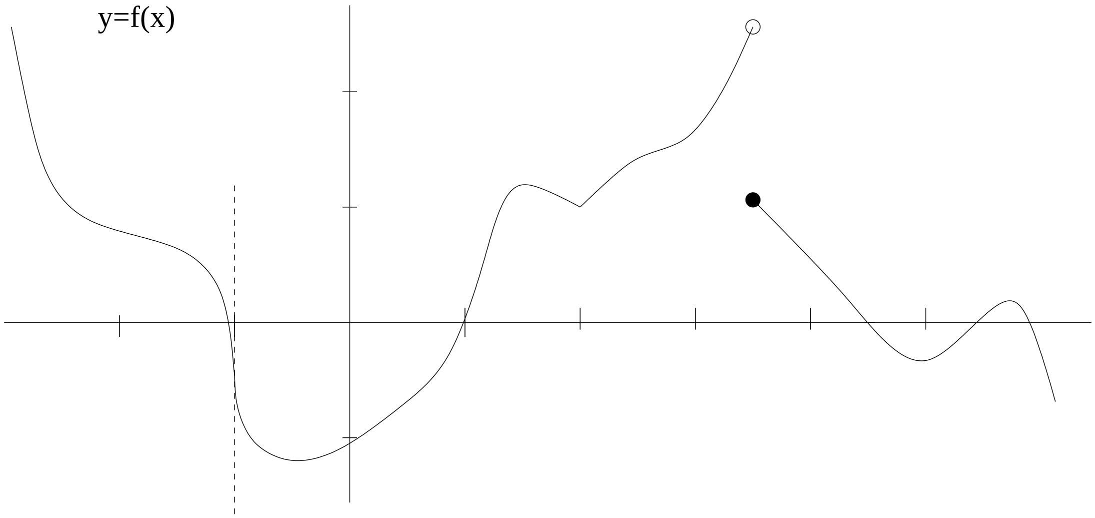
$$= \lim_{h\to 0} \frac{f(x+h)(g(x+h)-g(x)) + g(x)(f(x+h)-f(x))}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)(g(x+h)-g(x))}{h} + \frac{g(x)(f(x+h)-f(x))}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h)-f(x+h)g(x)+f(x+h)g(x)-f(x)g(x)}{h}$$

3. (10 pts) Graph of Derivative. The graph of the function $g(x)$ is given below. Sketch the graph of $g'(x)$ on the same coordinate axes.



4. (27 pts) Calculating Derivatives Using Differentiation Formulas.

(a) Find $f'(x)$. $\qquad f(x) = \dfrac{3}{4\sqrt{x}} + \dfrac{1}{5}\sin(4x^2) - e^{-x}$

(b) Find $f'(x)$. $\qquad f(x) = \ln(x(7x^2 + 2))$

(c) Find y'. $\qquad y = \dfrac{5\sqrt{x} - 6x^2 + 2}{x^4}$

5. (15 pts) Application position/velocity.

An object moves along a straight line and its position at time t is given by

$$s(t) = 2t^3 - 12t^2 - 30t$$

where s is measured in feet and t in seconds.

(a) Find the average velocity of the object over the time interval $0 \leq t \leq 2$ seconds.

(b) Find the velocity of the object at time $t = 0$.

(c) At what time(s) is the object at rest?

(d) Find the total distance the object travels over the interval $0 \leq t \leq 6$.

Note: You do not need to algebraically simplify your answer.

6. (13 pts) Find the equation of the line tangent to the ellipse

$$x^2 + xy + y^2 = 1$$

at the point $(0, -1)$.

Part B

1. (15 pts) Max/Min values of $f(x)$ on a Closed Interval.

Find the absolute maximum and absolute minimum values of

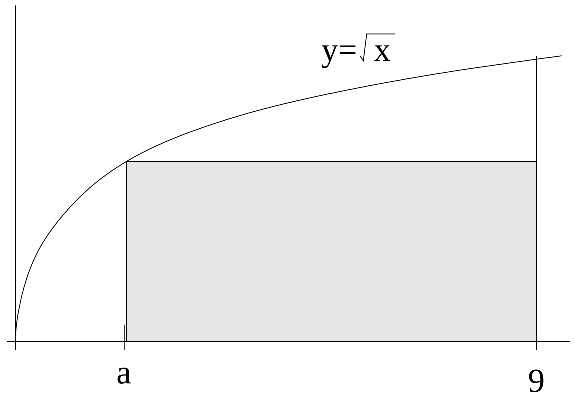
$$f(x) = 5xe^{-x}$$

on the interval $[0, 2]$.

The absolute minimum value is ______________________ which happens at ___________.

The absolute maximum value is ______________________ which happens at ___________.

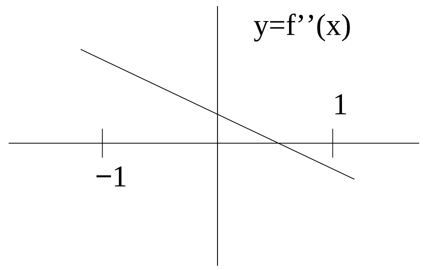
2. (18 pts) Optimization. The figure below shows a region bounded by the graphs of $y = \sqrt{x}, x = 9$,and $y = 0$. Suppose a rectangle with sides parallel to the axes is inscribed in the region bounded by the given curves and the left end of the rectangle is at $x = a$. Find the value of a and the dimensions of the resulting rectangle that has the maximum area.



Value of a that results in maximum area: __________

Dimensions of rectangle: __________ Maximum area: __________

3. (18 pts) Derivatives and the shape of the graph.
Consider the function f whose *second derivative* $f''(x)$ is graphed below.



Sketch a possible graph of f for each of the following conditions.

(i) f is increasing on $[-1, 1]$.

(ii) f is decreasing on $[-1, 1]$.

(iii) f has a local min at $x = 0$.

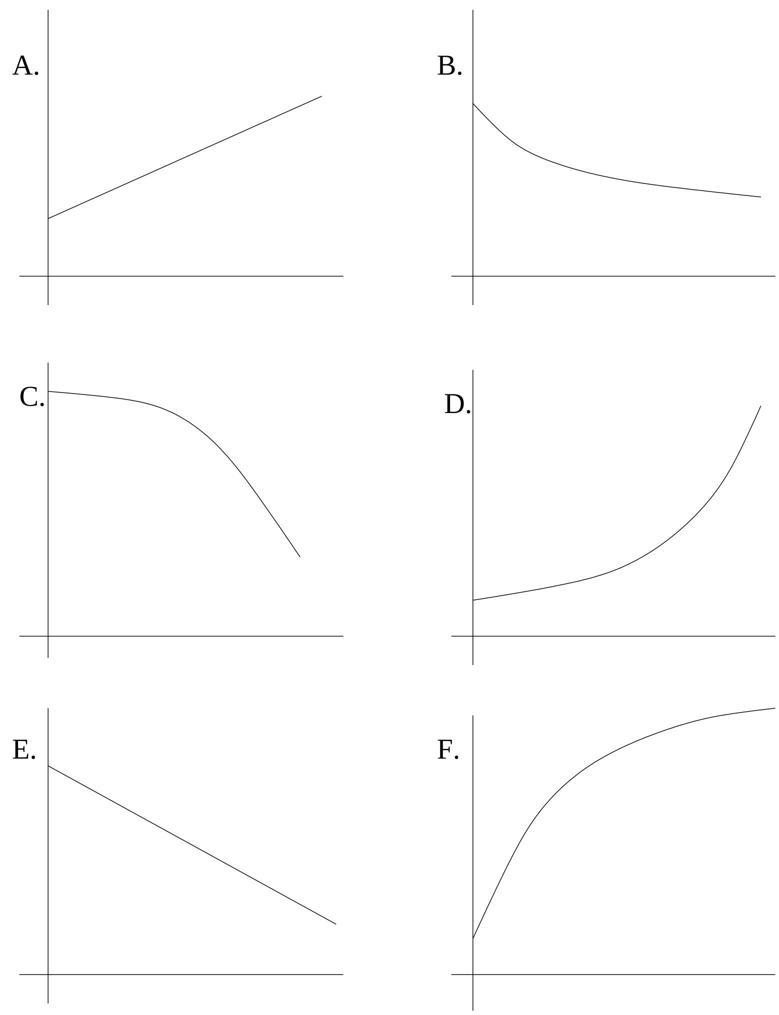
4. (16 pts) Biology Application. Graph matching. Match the following scenarios to the graph pictured below which most closely represents the **graph of the described population**. Write the letter of the matching graph in the space provided to the left of each scenario.

____________ A population of sea monkeys is increasing with an increasing growth rate.

____________ A population of wolves is growing with a decreasing growth rate.

____________ A population of hedgehogs has a constant positive growth rate.

____________ A population of killer whales is decreasing with an increasing growth rate.



5. (15 pts) Approximating an Integral using the Definition.

$$\int_0^6 \sqrt{x^2+1}\ dx$$

Approximate the integral above using a Riemann sum with left-hand endpoints and $n = 3$ subintervals.

If a positive function f function satisfies $f' > 0$ and $f'' > 0$ on [a,b], then is a Riemann sum with left-hand end points an overestimate or an underestimate of $\displaystyle\int_a^b f(x)\ dx$? Explain.

6. (18 pts) Calculating Integrals.

Calculate the following integrals. You do NOT have to algebraically simplify your solution.

(a) $\displaystyle\int_2^{10} 3x^4 - 2\sqrt{x} + \frac{x^3 - 4x^{1/3}}{x^2}\, dx$

(b) $\displaystyle\int 3 + 2\cos(2x)\, dx$

(c) $\displaystyle\int_1^6 f(x)\, dx$, where the graph of $f(x)$ is given below.