

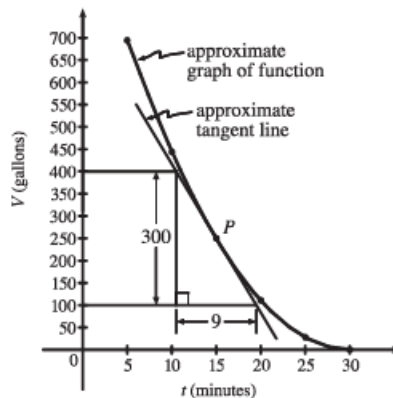
1. (a) Using  $P(15, 250)$ , we construct the following table:

$t$	$Q$	slope = $m_{PQ}$
5	(5, 694)	$\frac{694-250}{5-15} = -\frac{444}{10} = -44.4$
10	(10, 444)	$\frac{444-250}{10-15} = -\frac{194}{5} = -38.8$
20	(20, 111)	$\frac{111-250}{20-15} = -\frac{139}{5} = -27.8$
25	(25, 28)	$\frac{28-250}{25-15} = -\frac{222}{10} = -22.2$
30	(30, 0)	$\frac{0-250}{30-15} = -\frac{250}{15} = -16.\bar{6}$

(b) Using the values of  $t$  that correspond to the points closest to  $P$  ( $t = 10$  and  $t = 20$ ), we have

$$\frac{-38.8 + (-27.8)}{2} = -33.3$$

(c) From the graph, we can estimate the slope of the tangent line at  $P$  to be  $\frac{-300}{9} = -33.\bar{3}$ .



6. (a)  $y = y(t) = 10t - 1.86t^2$ . At  $t = 1$ ,  $y = 10(1) - 1.86(1)^2 = 8.14$ . The average velocity between times 1 and  $1 + h$  is

$$v_{\text{ave}} = \frac{y(1+h) - y(1)}{(1+h) - 1} = \frac{[10(1+h) - 1.86(1+h)^2] - 8.14}{h} = \frac{6.28h - 1.86h^2}{h} = 6.28 - 1.86h, \text{ if } h \neq 0.$$

(i)  $[1, 2]: h = 1, v_{\text{ave}} = 4.42 \text{ m/s}$

(ii)  $[1, 1.5]: h = 0.5, v_{\text{ave}} = 5.35 \text{ m/s}$

(iii)  $[1, 1.1]: h = 0.1, v_{\text{ave}} = 6.094 \text{ m/s}$

(iv)  $[1, 1.01]: h = 0.01, v_{\text{ave}} = 6.2614 \text{ m/s}$

(v)  $[1, 1.001]: h = 0.001, v_{\text{ave}} = 6.27814 \text{ m/s}$

(b) The instantaneous velocity when  $t = 1$  ( $h$  approaches 0) is  $6.28 \text{ m/s}$ .

1. As  $x$  approaches 2,  $f(x)$  approaches 5. [Or, the values of  $f(x)$  can be made as close to 5 as we like by taking  $x$  sufficiently close to 2 (but  $x \neq 2$ ).] Yes, the graph could have a hole at  $(2, 5)$  and be defined such that  $f(2) = 3$ .

2. As  $x$  approaches 1 from the left,  $f(x)$  approaches 3; and as  $x$  approaches 1 from the right,  $f(x)$  approaches 7. No, the limit does not exist because the left- and right-hand limits are different.

4. (a)  $\lim_{x \rightarrow 0} f(x) = 3$

(b)  $\lim_{x \rightarrow 3^-} f(x) = 4$

(c)  $\lim_{x \rightarrow 3^+} f(x) = 2$

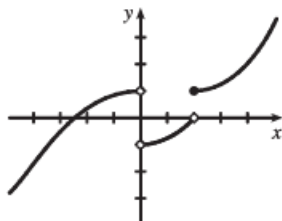
(d)  $\lim_{x \rightarrow 3} f(x)$  does not exist because the limits in part (b) and part (c) are not equal.

(e)  $f(3) = 3$

8. (a)  $\lim_{x \rightarrow 2} R(x) = -\infty$                       (b)  $\lim_{x \rightarrow 5} R(x) = \infty$                       (c)  $\lim_{x \rightarrow -3^-} R(x) = -\infty$
- (d)  $\lim_{x \rightarrow -3^+} R(x) = \infty$
- (e) The equations of the vertical asymptotes are  $x = -3$ ,  $x = 2$ , and  $x = 5$ .

10.  $\lim_{t \rightarrow 12^-} f(t) = 150$  mg and  $\lim_{t \rightarrow 12^+} f(t) = 300$  mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at  $t = 12$  h. The left-hand limit represents the amount of the drug just before the fourth injection. The right-hand limit represents the amount of the drug just after the fourth injection.

14.  $\lim_{x \rightarrow 0^-} f(x) = 1$ ,  $\lim_{x \rightarrow 0^+} f(x) = -1$ ,  $\lim_{x \rightarrow 2^-} f(x) = 0$ ,  
 $\lim_{x \rightarrow 2^+} f(x) = 1$ ,  $f(2) = 1$ ,  $f(0)$  is undefined



18. For  $f(x) = \frac{x^2 - 2x}{x^2 - x - 2}$ :

$x$	$f(x)$	$x$	$f(x)$
0	0	-2	2
-0.5	-1	-1.5	3
-0.9	-9	-1.1	11
-0.95	-19	-1.01	101
-0.99	-99	-1.001	1001
-0.999	-999		

It appears that  $\lim_{x \rightarrow -1} \frac{x^2 - 2x}{x^2 - x - 2}$  does not exist since

$f(x) \rightarrow \infty$  as  $x \rightarrow -1^-$  and  $f(x) \rightarrow -\infty$  as  $x \rightarrow -1^+$ .

21. For  $f(x) = \frac{\sqrt{x+4}-2}{x}$ :

$x$	$f(x)$	$x$	$f(x)$
1	0.236068	-1	0.267949
0.5	0.242641	-0.5	0.258343
0.1	0.248457	-0.1	0.251582
0.05	0.249224	-0.05	0.250786
0.01	0.249844	-0.01	0.250156

It appears that  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x} = 0.25 = \frac{1}{4}$ .

28.  $\lim_{x \rightarrow 0} \frac{x-1}{x^2(x+2)} = -\infty$  since  $x^2 \rightarrow 0$  as  $x \rightarrow 0$  and  $\frac{x-1}{x^2(x+2)} < 0$  for  $0 < x < 1$  and for  $-2 < x < 0$ .