Math 141, Test 2, practice problems

1. (`Definition of derivative`) Use the definition of the derivative f' to find derivative of

   (a) $f(x) = \sqrt{x}$ at the given $x = 1$.

   (b) $f(x) = x^3$ at $x = 2$.

   (c) $f(x) = \frac{1}{x^2}$ at the given $x = 1$.

   (d) $f(x) = \sqrt{4x+3}$ at any $x > -\frac{3}{4}$.

2. (`A meaning of derivative, slope of tangent line`) Consider the graph of $f(x) = x^3$. Find the equation of the tangent line at the point $(2, 8)$ on the graph.

3. (`Differentiabilty implies continuity. But, not the converse`) Find an example of function which is continuous at $x = 0$, but not differentiable at $x = 0$.

4. (`Power rule`) Use power rules to find the deriavtives f' of

   (a) $f(x) = x^3 - x^2 - 5x + 8 - \frac{1}{x^5}$.

   (b) $f(x) = 3x^5 + \frac{2}{\sqrt{x}}$.

   (c) $f(x) = 1 - x^2 + 3x^3\sqrt{x}$.

5. (`Derivative, rate of change, velocity`) On top of a cliff with height $73.5\ m$ we throw a ball upward. The height of the ball from the ground follows the function $h(t) = 73.5 + 9.8t - 4.9t^2$.

   (a) Find the maximum height the ball can reach.

   (b) Find the velocity when the ball hits the ground.

6. (`Derivative, rate of change, velocity`) A particle is moving along a line with displacement function $s(t) = t^3 - 6t^2 + 9t$.

   (a) When are the moments the particle change its direction.

   (b) Find the total distance the particle traveled in $t = 4$.

7. (`Derivative of` e^x) Find the derivative f' of the function $f(x) = e^{x+1} - 2x + \ln 2$.

8. (`Product rule, Quotient rule`) Find the deriavtives f' of the following functions.

   (a) $f(x) = (x^2 - 3x + 1)(3x + 2)$.

   (b) $f(x) = e^x(\sqrt{x} + 5x^3)$.

   (c) $f(x) = \frac{x+2}{\sqrt{x}-2}$ at $x = 1$.

   (d) $f(x) = \frac{e^x}{3-x}$.

9. (`Derivatives of trigonometric functions`) Find the derivatives f' of the following functions.

   (a) $f(x) = \sin x + \tan x - \sec x$.

   (b) $f(x) = \sec x \tan x \quad$ at $x = \frac{\pi}{3}$.

   (c) $f(x) = xe^x \sin x$.

   (d) $f(x) = \frac{\tan x - 1}{\sec x} \quad$ at $x = \frac{\pi}{3}$.

10. (`An important limit`) Taking $f(x) = \sin x$, we observed

$$1 = \cos 0 = f'(0) = \lim_{x \to 0} \frac{\sin x - \sin 0}{x - 0} = \lim_{x \to 0} \frac{\sin x}{x}.$$

    Use above identity to find the limit of

$$\lim_{x \to 0} \frac{\sin(4x)}{3x}.$$

11. (`Chain rule`) Find the derivatives f' of the following functions.

    (a) $f(x) = (3x^3 - 2x^2 + 5)^{331}$.

    (b) $f(x) = \sqrt{1 - x^2}$.

    (c) $f(x) = \sin(x^3)$.

    (d) $f(x) = \sec^2 x - \tan^2 x$.

    (e) $f(x) = (\sin(\tan x))^3$.

    (f) $f(x) = e^{\cos x}$.

    (g) $f(x) = 2^{-x^2}$.

    (h) $f(x) = x \sin\left(\frac{1}{x}\right)$ for $x > 0$.

12. (`Implicit differentiation`) Consider the graph of $\sqrt{x} + \sqrt{y} = 1$ in the xy-plane. Find the equation of the tangent line at the point $(\frac{1}{4}, \frac{1}{4})$ on the graph.

13. (`Implicit differentiation`) Find the equation of the tangent line to $x^3 + y^3 = 4xy$ at the point $(2, 2)$.

14. (`Implicit differentiation`) Find the equation of the tangent line to $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at the point $(3, 1)$.

15. (`Shapes of the inverse trigonometirc functions`) Sketch the graph of inverse trigonometric functions $y = \sin^{-1} x,\ y = \cos^{-1} x,\ y = \tan^{-1} x$. What are the domains and ranges.

16. (`Derivatives of inverse trigonometric functions`) Find the derivatives f' of the following functions.

    (a) $f(x) = x \arctan \sqrt{x}$.

    (b) $f(x) = \sin^{-1}(x^3)$.

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⟨Answer keys⟩

1. (a) $f'(1) = \frac{1}{2}$ (b) $f'(2) = 12$ (c) $f'(1) = -2$ (d) $f'(x) = \frac{2}{\sqrt{4x+3}}$
2. $y = 12x - 16$
3. $f(x) = |x|$
4. (a) $f'(x) = 3x^2 - 2x - 5 + \frac{5}{x^6}$ (b) $f'(x) = 15x^4 - \frac{1}{x^{3/2}}$ (c) $f'(x) = -2x + \frac{21}{2}x^{5/2}$
5. (a) $78.4\ m$ (b) $-39.2\ m/s$
6. (a) $t = 1,\ \ t = 3$ (b) 12
7. $f'(x) = e^{x+1} - 2$
8. (a) $f'(x) = 9x^2 - 14x - 3$ (b) $f'(x) = e^x(\sqrt{x} + 5x^3 + \frac{1}{2\sqrt{x}} + 15x^2)$ (c) $-\frac{5}{2}$ (d) $f'(x) = \frac{e^x(4-x)}{(3-x)^2}$
9. (a) $f'(x) = \cos x + \sec^2 x - \sec x \tan x$ (b) $f'(\frac{\pi}{3}) = 14$
   (c) $f'(x) = e^x(\sin x + x \sin x + x \cos x)$ (d) $f'(\frac{\pi}{3}) = \frac{1+\sqrt{3}}{2}$
10. $\frac{4}{3}$
11. (a) $f'(x) = 331(3x^3 - 2x^2 + 5)^{330}(9x^2 - 4x)$ (b) $f'(x) = -\frac{x}{\sqrt{1-x^2}}$ (c) $f'(x) = 3x^2 \cos(x^3)$
    (d) $f'(x) = 0$ (e) $f'(x) = 3(\sin(\tan x))^2 \cos(\tan x) \sec^2 x$ (f) $f'(x) = -\sin x e^{\cos x}$
    (g) $f'(x) = -2x2^{-x^2}$ (h) $f'(x) = \sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right)$
12. $y = -x + \frac{1}{2}$
13. $y = -x + 4$
14. $y = -\frac{9}{13}x + \frac{40}{13}$
15. Lecture note
16. (a) $f'(x) = \arctan\sqrt{x} + \frac{\sqrt{x}}{2(1+x)}$ (b) $f'(x) = \frac{3x^2}{\sqrt{1-x^6}}$