MATH 142 Midterm Exam #1

October 4, 2004 10:20AM, Prof Johnson

NAME:_____

- **No calculators are allowed on this exam.**
- **Answers such as $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ are perfectly fine!! However you MUST simplify expressions such as $\sin(\pi/3)$.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please include all information about the u-substitutions or integration by parts choice(s) that you make.**

$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$	$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$	$\sin(2x) = 2\sin(x)\cos(x)$

Problem	Points	Score
1 (a)(b)	30	
1 (c)	15	
2	15	
3	25	
4	15	
total	100	

1. Integrals. [45 points, 15 each]

Evaluate the integrals. Show all work and include all information about substitutions and integration by parts choices, and restrictions on angles for substitutions etc.

(a) $\displaystyle\int_0^1 e^{6x+2}\,dx$

(b) $\displaystyle\int_1^2 x^4 \ln(3x)\,dx$

(c) $\int_0^1 3\sin(\frac{\pi x}{4})\ dx$

2. The velocity of a tug boat is given by $v(t) = 5t^{-1} + \sqrt{t^5}$ meters per minute, for $1 \leq t \leq 10$ minutes. Find the average velocity of the boat over the time interval $t = 2$ to $t = 6$ minutes.

3. Volume. [25 points]

Note: On this problem, you can earn partial credit for parts (b)-(e) by sketching an arbitrary slice of the volume in the space provided in the left margin.

Consider the region, R which is bounded between the curves

$y = x^2 + 4x + 4$ $\qquad$ $y = -x + 4$

(a) Sketch the region.
Label the points of intersection.

(b) Write an integral for the volume of the solid formed by rotating this region R about the line x=2. DO NOT evaluate the integral.

(c) Now consider the solid whose base is the region R and whose cross-sections above the xy-plane and perpendicular to the x-axis (i.e. slices parallel to the y-axis) are squares. Write an integral for the volume of this solid, but DO NOT evaluate the integral.

4. Work. [15 points]

The tank pictured below is filled with oil to a depth of 2 meters. You may assume the oil has a density of 1380 kg/m^3. Write BUT DO NOT EVALUATE the integral which calculates the work required to pump the oil out of the outlet on the top of the tank. Note: the acceleration due to gravity is 9.8 m/sec^2.