MATH 162

Final Exam QUESTIONS

December 16, 2003

Part A

1. (30 points) Integrals. Evaluate the following integrals if they exist. If an integral does not exist, show why it diverges.

(i) $\displaystyle\int \sin^{52} x \cos x \, dx$

(ii) $\displaystyle\int_0^2 \sqrt{4-x^2}\, dx$

(iii) $\displaystyle\int_1^2 \frac{1}{(x-1)^{3/2}}\, dx$

(iv) $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^3}\, dx$

(v) $\displaystyle\int \frac{x^4}{x^4-1}\, dx$

2. (20 points) Volumes and Areas. Consider the region between the curves $y = x^2 - 4x$ and $y = x - 4$.

(a) Sketch the curves given above, label the points of intersection, and shade the region **between the curves**. Be careful.

(b) Set up but do not evaluate an integral for the *volume* of revolution generated by revolving the region about the x–axis.

(c) Set up but do not evaluate an integral for the *volume* of revolution generated by revolving the region about the line $x = -2$. Be careful, where is the line $x = -2$?

(d) Sketch the portion of the curve $y = x^2 - 4x$ which is below the x-axis. Set up but do not evaluate an integral for the area of the *surface* of revolution generated by revolving the portion of the curve $y = x^2 - 4x$ which is below the x-axis about the x-axis.

3. (5 points) Parametric equations and Area.

The ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

can be parameterized by $x = a\cos(t)$ and $y = b\sin(t)$. Use this parameterization to find the area inside this ellipse. Your answer should be a formula in terms if a and b.

4. (21 points) Polar Curves, Graph matching.

Write the letter of the graph that best fits the graph of the given *polar curves* in the space provided. For all curves, $0 \le \theta \le 2\pi$.

(i) $r = \theta$ ______________________

(ii) $r = 2\sin(\theta)$ ______________________

(iii) $r = 2\cos(\theta)$ ______________________

(iv) $\theta = \frac{\pi}{4}$ ______________________

(v) $r = \frac{\pi}{4}$ ______________________

(vi) $r = 1 + \cos(\theta)$ ______________________

(vii) $r = 1 + \sin(\theta)$ ______________________

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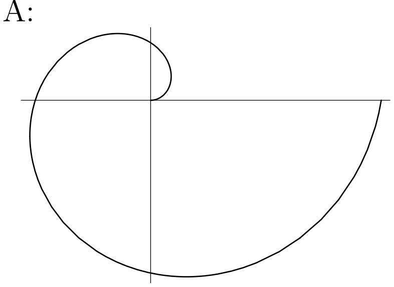

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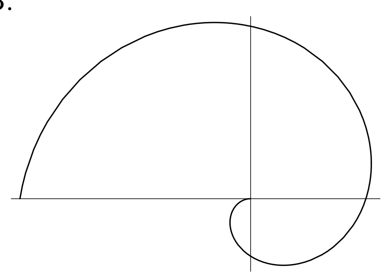

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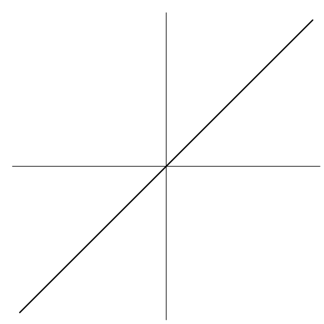

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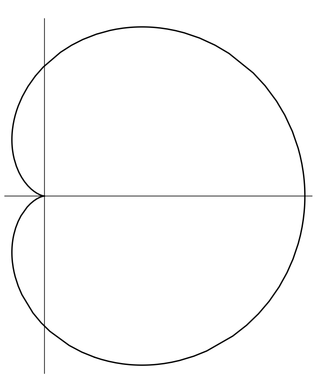

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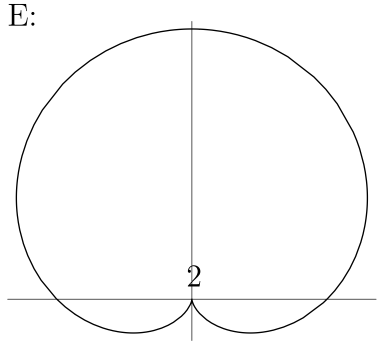

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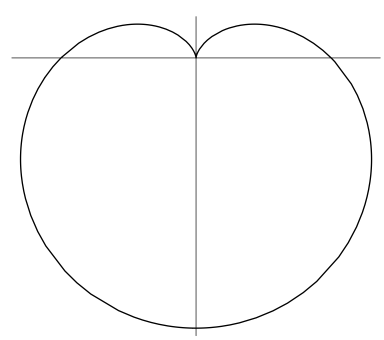

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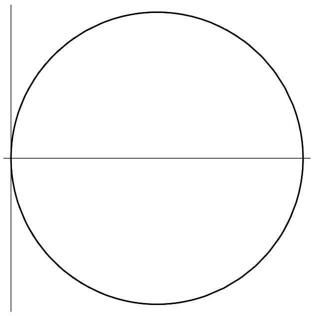

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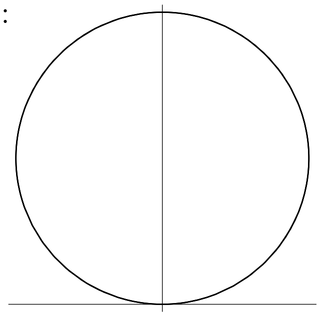

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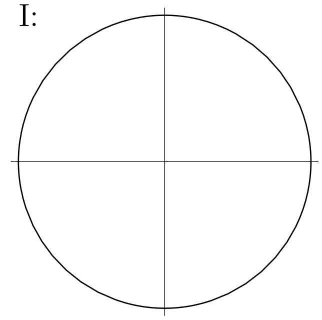


5. (24 points) Parametric Curves, Graph matching. Write the letter of the graph that best fits the graph of the given *parametric curves* in the space provided.

(i) $\begin{cases} x &= 5t \\ y &= -(5t)^2 + 1 \end{cases}$ ____________

(ii) $\begin{cases} x &= 5t^2 \\ y &= -(5t^2)^2 + 1 \end{cases}$ ____________

(iii) $\begin{cases} x &= \sin(t) \\ y &= -\sin^2(t) + 1 \end{cases}$ ____________

(iv) $\begin{cases} x &= \cos(t) \\ y &= \sin(t)\cos(t) \end{cases}$ ____________

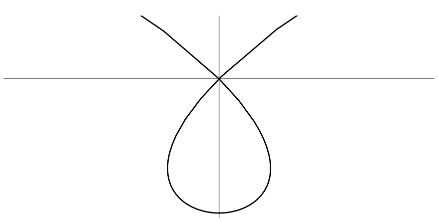
(v) $\begin{cases} x &= t^2 - 1 \\ y &= t^3 - t \end{cases}$ ____________

(vi) $\begin{cases} x &= t^3 - t \\ y &= t^2 - 1 \end{cases}$ ____________

(vii) $\begin{cases} x &= \sin(2t) \\ y &= \sin(3t) \end{cases}$ ____________

(viii) $\begin{cases} x &= t + \sin(2t) \\ y &= t + \sin(3t) \end{cases}$ ____________

A:



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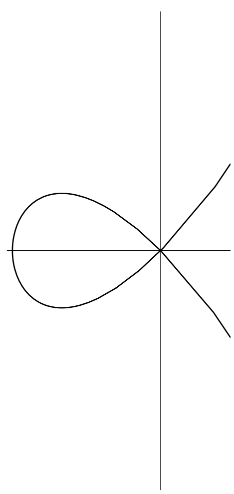

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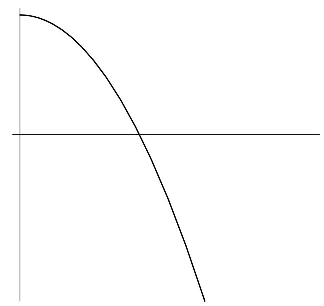

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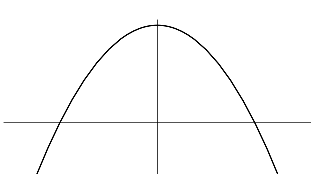

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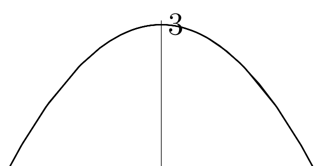

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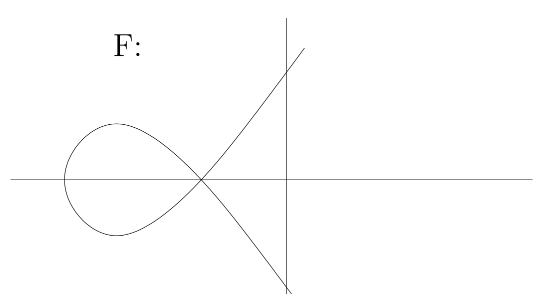

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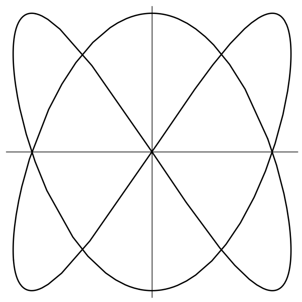

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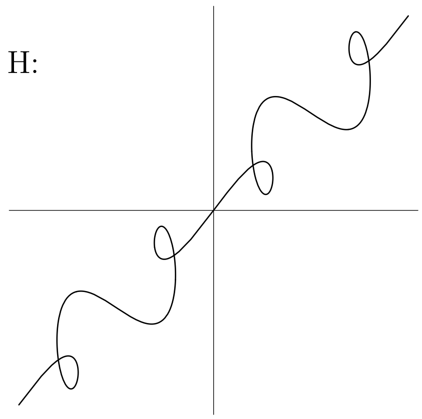

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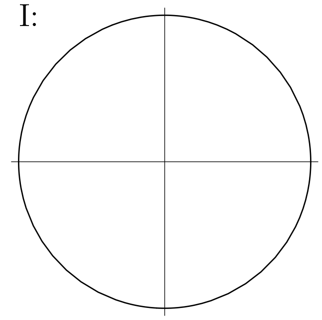

Part B

1. (24 points) Sequences and series. Determine whether the statements are TRUE or FALSE. True means always true, false means sometimes false.

(a) ______________ The **sequence** $\{\frac{3n}{n+1}\}_{n=1}^{\infty}$ converges.

(b) ______________ The sequence $\{1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, \cdots\}$ converges to 0.

(c) ______________ $1 - 1 + 1 - 1 + 1 - 1 + 1 - \ldots$ converges to zero.

(d) ______________ If $\sum |a_n|$ converges and $\lim_{n\to\infty} |\frac{a_n}{b_n}| = 1$, then the series $\sum |b_n|$ converges.

(e) ______________ If $\sum a_n$ diverges then $\sum |a_n|$ diverges.

(f) ______________ The series $50 + 10 + 2 + \frac{2}{5} + \frac{2}{25} + \cdots$ is a geometric series which converges to $\frac{125}{2}$

(g) ______________ If the power series $\sum c_n(x-4)^n$ converges for $x = 8$ then the series $\sum c_n(3^n)$ is convergent.

(h) ______________ $$\frac{1}{e} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

2. (36 points) Determine whether the following series converge or diverge. Justify your answer using one of the series tests. You *must* state the name of the test you use and indicate why your answer follows from this test.

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{2n-1}}$

(b) $\sum_{k=1}^{\infty} ke^{-k^2}$

(c) $\sum_{n=1}^{\infty} \ln(\frac{n}{2n+5})$

(d) $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{2n+n^3+1}$

(e) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$

(f) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$

3. (14 points) Power Series, Interval of Convergence.

Consider the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n(x-1)^n}{5^n\sqrt{n}}$$

a) Find the radius of convergence for the power series.

b) What is the interval of convergence? Justify your answer by checking the endpoints.

4. (13 points) Using Power Series to Calculate Integrals.

(a) Find the Maclaurin series for the function $f(x) = e^{x^3}$.

(b) Use the power series above to calculate the integral $\int_{-1}^{0} e^{x^3} dx$. (Your answer will be a series.)

(c) If you approximate your answer to part (b) by summing the first four terms of the series, give a bound for your error according to the Alternating Series Estimation Theorem,

5. (13 points) Taylor Series

(a) Find the degree 3 Taylor polynomial, $T_3(x)$, for $f(x) = \sqrt[3]{x}$ centered at $a = 27$.

b) Use $T_3(x)$ to approximate $\sqrt[3]{26}$ and give the best bound possible on the error according to Taylor's Approximation Theorem. Note, the Taylor's Approximation Theorem is given below.

Taylor's Approximation Theorem:

If $|f^{(k+1)}(x)| \leq M$ for $|x - a| \leq d$,

then $|R_k(x)| \leq \dfrac{M}{(k+1)!}|x - a|^{k+1}$ for $|x - a| \leq d$.