MATH 162

Midterm 2

November 4, 2003

NAME (please print legibly): _____

Your University ID Number: _____

Circle your Instructor's Name:

Inga Johnson Steve Gonek

- **No calculators are allowed on this exam.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please put your final answers in the spaces provided.**
- **When integrating, put down all information you are using, such as substitutions or integration by parts.**
- **Simplify expressions such as** $\sin(\pi/3)$

Formulas:

$$\sin^2 x = \frac{1}{2}\big(1 - \cos(2x)\big) \qquad \cos^2 x = \frac{1}{2}\big(1 + \cos(2x)\big)$$

Expression	Substitution	
$\sqrt{a^2 - x^2}$	$x = a\sin\theta,$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta,$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta,$	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

QUESTION	VALUE	SCORE
1	24	
2	15	
3	30	
4	17	
5	14	
TOTAL	100	

1. (24 points) S etch the graph of each of the parametric curves below on the axes provided. Label each curve with arrows indicating the direction of travel for increasing time, t, or describe it in words.

(a) $\begin{cases} x = \cos^2(t) \\ y = \sin^2(t) \end{cases}$

(b) $\begin{cases} x = t \\ y = 1 - t \end{cases}$

(c) $\begin{cases} x = 2t^2 \\ y = 1 - 2t^2 \end{cases}$

2. (15 points) i l in the blank with the correct statement of the comparison statement.

Comparison Theorem for Integrals:

Suppose that f and g are continuous functions with $0 \leq f(x) \leq g(x)$ for $x \geq a$.

(i) If $\int_a^\infty$ _____ dx is convergent, then so is $\int_a^\infty$ _____ dx.

(ii) If $\int_a^\infty$ _____ dx is divergent, then so is $\int_a^\infty$ _____ dx.

Consider the integral $\displaystyle\int_1^\infty \frac{x}{\sqrt[5]{x^{15}+5x^3+1}}\,dx$

Some of the inequalities below are true and some are false. Choose the inquality from the list below which is true AND can be used with the comparison theorem to deduce whether or not the integral above is convergent or divergent. Write your answer and conclusion in the blank spaces provided below.

1. $\displaystyle\frac{x}{\sqrt[5]{x^{15}+5x^3+1}} \leq \frac{1}{x}$
2. $\displaystyle\frac{x}{\sqrt[5]{x^{15}+5x^3+1}} \geq \frac{1}{x}$
3. $\displaystyle\frac{x}{\sqrt[5]{x^{15}+5x^3+1}} \leq \frac{1}{x^{\frac{1}{3}}}$
4. $\displaystyle\frac{x}{\sqrt[5]{x^{15}+5x^3+1}} \geq \frac{1}{x^{\frac{1}{3}}}$
5. $\displaystyle\frac{x}{\sqrt[5]{x^{15}+5x^3+1}} \leq \frac{1}{x^2}$
6. $\displaystyle\frac{x}{\sqrt[5]{x^{15}+5x^3+1}} \geq \frac{1}{x^2}$
7. $\displaystyle\frac{x}{\sqrt[5]{x^{15}+5x^3+1}} \leq \frac{1}{x^5}$
8. $\displaystyle\frac{x}{\sqrt[5]{x^{15}+5x^3+1}} \geq \frac{1}{x^5}$

Using inequality number _____ and the Comparison Theorem for Integrals we can deduce that the integral $\displaystyle\int_1^\infty \frac{x}{\sqrt[5]{x^{15}+5x^3+1}}\,dx$ is

CONVERGENT. DIVERGENT. (circle one)

3. (30 points) u tiple choice

1. Evaluate the improper integral

$$\int_1^\infty \frac{\ln x}{x^3}\, dx.$$

a) $1/4$
b) $1/3$
c) $1/2$
d) 1
e) $\ln 2$
f) $\ln 3$
g) $\ln 4$
h) diverges

2. Evaluate the integral

$$\int_0^{1/2} \frac{x^2}{x^2-1}\, dx.$$

a) $1/2(1-\ln 3)$
b) $\ln 3$
c) $1-\ln 3/4$
d) $1+\ln 3/4$
e) $1+\ln 3$
f) $-1/2\ln 3$
g) $1-\ln 3$
h) $1/2\ln 3$

4. (17 points) A urve is defined by the parametric equations

$$x = t^2 + 1 \qquad \text{and} \qquad y = \sqrt{t-1}\,.$$

Find the slope of the tangent line to the curve at the point $(26, 2)$.

5. (14 points) A urve is defined parametrically by the equations

$$x = 4t \qquad \text{and} \qquad y = 1/2t^2\,.$$

Set up, BUT DO NOT EVALUATE, an integral for the length of the curve between the points $(0, 0)$ and $(12, 9/2)$.

$$\int_{\square}^{\square} \square \; dx$$