MATH 162

FINAL

May 6, 2002

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Circle your Instructor's Name:

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- **The first part of the final can replace your lowest midterm score, but it will also count towards your score on the final. If you skip it, you will get at most 100 points out of 200.**
- **No calculators are allowed on this exam.**
- **Please put your final answers in the spaces provided.**
- **When integrating, put down all information you are using, such as substitutions or integration by parts.**

Part A		
QUESTION	VALUE	SCORE
1	33	
2	15	
3	11	
4	30	
5	11	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
6	20	
7	10	
8	30	
9	14	
10	10	
11	16	
TOTAL	100	

Part A

Formulas:

$$\sin^2 x = \frac{1}{2}\big(1-\cos(2x)\big)$$
$$\cos^2 x = \frac{1}{2}\big(1+\cos(2x)\big)$$

Expression	Substitution		Identity
$\sqrt{a^2-x^2}$	$x=a\sin\theta,$	$-\pi/2\le\theta\le\pi/2$	$1-\sin^2\theta=\cos^2\theta$
$\sqrt{a^2+x^2}$	$x=a\tan\theta,$	$-\pi/2<\theta<\pi/2$	$1+\tan^2\theta=\sec^2\theta$
$\sqrt{x^2-a^2}$	$x=a\sec\theta,$	$0\le\theta<\pi/2$ or $\pi\le\theta<3\pi/2$	$\sec^2\theta-1=\tan^2\theta$

1. (33 pts) Evaluate the following integrals.

(a) (11 points)

$$\int x^3 e^{2x^2}\,dx$$

(b) (11 points)

$$\int \sin^3 x \cos^3 x \; dx$$

ANSWER: ____________________________

(c) (11 points)

$$\int_1^4 \frac{1}{\sqrt[3]{x^2 - 4x + 4}} \; dx$$

2. (15 pts) Find the volume the solid generated by rotating the region bounded by the x-axis, the y-axis, $y = 2$, and the curve

$$y = x^3 + x$$

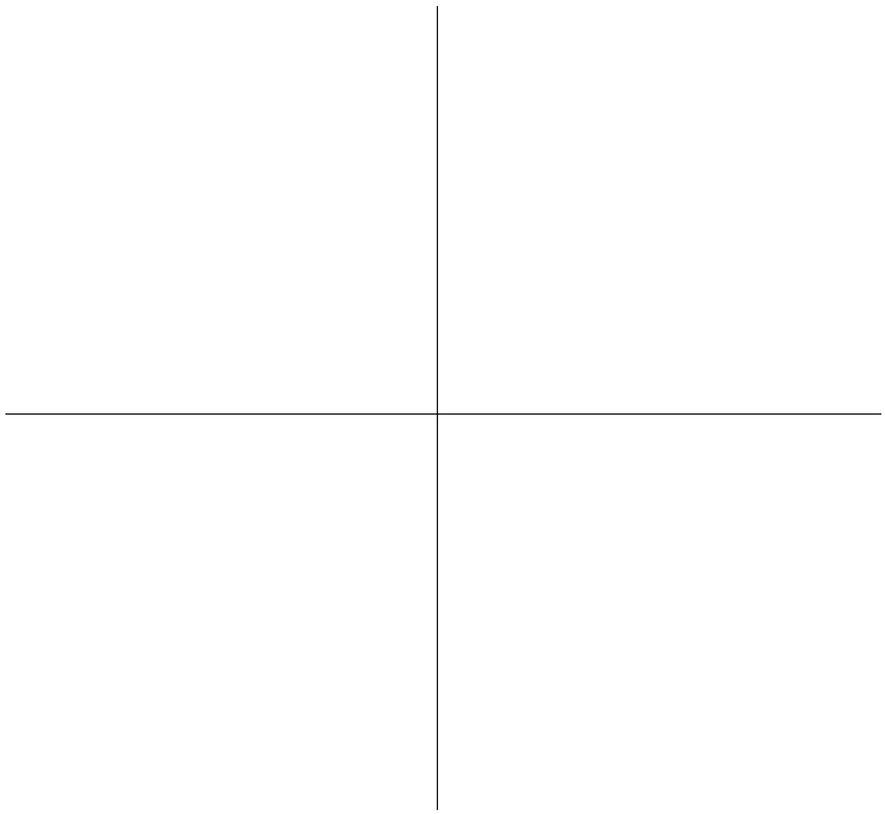
around the y-axis.

3. (11 pts) If the work needed to stretch a spring 2 feet beyond its natural length is 15 foot-pounds, how much work is needed to stretch it 3 feet beyond its natural length?

4. (30 pts)

(a) (9 points) Sketch the graph of the polar equation $r = 2(1 + \cos(\theta))$.

(b) (9 points) On the same set of axes, sketch the graph of $r = 6\cos(\theta)$. Be sure to label which graph represents which function.



(c) (12 points) Find the area of the region lying inside of $r = 6\cos(\theta)$ and outside of $r = 2(1 + \cos(\theta))$.

5. (11 pts) Consider the curve given by the parametric equations

$$x = 1 - t^2, \quad y = \sin t \quad (-\pi \leq t \leq \pi).$$

With this range of values for t, the curve is a loop traced out exactly once.

Find the slope of the curve at the point $\left(1 - \frac{\pi^2}{36}, -\frac{1}{2}\right)$.

Part B

Theorem If $\|f^{(n+1)}(x)\| \le M$ for $\|x-a\| \le d$, then the remainder $R_n(x)$ of the Taylor series satisfies the inequality $$\|R_n(x)\| \le \frac{M}{(n+1)!}\|x-a\|^n \qquad \text{for } \|x-a\| \le d.$$

6. (20 pts) (Each part is worth 2 points) Determine whether the statements are true or false.

(a) The **sequence** $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ diverges since $\lim_{n\to\infty} \frac{n}{n+1} = 1$.

ANSWER: ____________________

(b) If a sequence $\{a_n\}$ is monotone and increasing then $\lim_{n\to\infty} a_n = \infty$.

ANSWER: ____________________

(c) The sequence $\{0, 2, 0, 2, 0, \ldots\}$ diverges

ANSWER: ____________________

(d) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

ANSWER: ____________________

(e) If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} (a_n + a_{n+1})$ converges.

ANSWER: ____________________

(f) If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} \sin(n) a_n$ converges.

ANSWER: ____________________

(g) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 2$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

ANSWER: ____________________

(h) If $\sum_{n=1}^{\infty} |a_n|$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges.

(j) The series $\sum_{n=1}^{\infty} \frac{10^n}{8^n}$ converges since $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{10}{8}$.

ANSWER: ____________________

7. (10 pts) Determine whether the series converges or diverges.
If it converges find its **sum**.

$$\sum_{n=1}^{\infty} \frac{3^{n-4}}{2^{2n}}$$

8. (30 pts) Determine whether the series converges or diverges. State the convergence tests you are using, and check the hypotheses of the tests.

(a) (6 points)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \ln(n)}$$

ANSWER: ______________________

(b) (6 points)

$$\sum_{n=1}^{\infty} \frac{n}{e^n}$$

(c) (6 points)

$$\sum_{n=1}^{\infty} \frac{3n+1}{\pi - n}$$

ANSWER: ____________________

(d) (6 points)

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^3}$$

ANSWER: ____________________

(e) (6 points)

$$\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$$

9. (14 pts) Find the radius and interval of convergence for the following series.

(a) (7 points)

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n$$

ANSWER: ____________________

(b) (7 points)

$$f(x) = \sum_{n=1}^{\infty} \frac{n!}{(2n)!}x^n$$

10. (10 pts) Find a power series representation, centered at 0, for the function below, and determine its **interval** of convergence.

$$f(x) = \frac{x}{4x + 1}$$

11. (16 pts)

(a) (8 points) Find the 4^{th} degree Taylor polynomial, $T_4(x)$, for $f(x) = \sqrt{x}$ centered at $a = 9$. You do **NOT** need to simplify your answer by multiplying out the fractions.

ANSWER: ____________________

(b) (8 points) Use the 4^{th} degree Taylor polynomial, $T_4(x)$, to approximate $\sqrt{9.1}$ and give a bound on the error of this approximation.