MATH 142 Midterm Exam #1

October 6, 2006

NAME:____

- **No calculators are allowed on this exam.**
- **Answers such as** $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ **are perfectly fine!! However you MUST simplify expressions such as** $\sin(\pi/3)$**.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please include all information about u-substitutions, and use correct mathematical grammar in the presentation of your solution.**

Problem	Points	Score
1	20	
2	30	
3	15	
4	15	
5	20	
total	100	

$$\sum_{i=1}^{n} a = a \cdot n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

1. Definition of the Integral. Recall the definition of the definite integral for a continuous function $f(x)$ on the interval $[a, b]$.

$$\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Calculate the integral below using the definition of the integral.

$$\int_0^3 5 - x^2\ dx$$

(a) First, find the following quantities:

$\Delta x =$ ____________ $\quad x_i =$ ____________ $\quad f(x_i) =$ ________________________

(b) Next, using the quantities above and the summation formulas on the front page of the exam, simplify $\sum_{i=1}^{n} f(x_i)\Delta x$ into an expression without the summation notation.

(c) Last, evaluate the limit, $\lim_{n\to\infty}(\sum_{i=1}^{n} f(x_i)\Delta x)$.

Note: you can *check* your answer by using the Fundamental Theorem of Calculus.

2. Integrals. Evaluate the following definite and indefinite integrals.

(a) $\displaystyle\int (t^2 - 7\sqrt{t} + 45t)t^{-2}\ dt$

(b) $\displaystyle\int x\sqrt{5-x}\ dx$

(c) $\displaystyle\int_0^{4/3} \frac{3}{9x^2+16}\ dx$

3. Let $f(t)$ be the continuous function graphed below and let $g(x) = \int_{-3}^{x} f(t)\, dt$

(a) Evaluate $g(4)$.

(b) Find $g'(x)$ and evaluate $g'(0)$.

(c) Find the intervals over which $g(x)$ is concave up.

4. Evaluate the following limits. Show work and indicate if/when L'Hôpital's rule is used.

(a) $\lim_{x \to 0} \dfrac{1 - \cos(4x)}{1 - \cos(3x)}$

(b) $\lim_{x \to \infty} x^2 \ln(x^2 + x)$

5. Volume. Note: On this problem, you can earn partial credit for parts (b)-(e) by sketching an arbitrary slice of the volume in the space provided in the left margin.

Consider the region, R which is bounded between the curves

$$y = 5 \qquad y = x + 1 \qquad y = -2x + 7$$

(a) Sketch the region and label the points of intersection.

(b) Write an integral for the volume of the solid formed by rotating the region R about the y-axis. DO NOT evaluate the integral.

(c) Write an integral for the volume of the solid formed by rotating the region R about the line $y = -1$. DO NOT evaluate the integral.

(d) Now consider the solid whose base is the region R and whose cross-sections above the xy-plane and perpendicular to the y-axis (i.e. slices parallel to the x-axis) are squares. Write an integral for the volume of this solid, but DO NOT evaluate the integral.