MATH 142 Midterm Exam #1

November 5, 2004 10:20AM, Prof Johnson

NAME:_____

- **No calculators are allowed on this exam.**
- **Answers such as $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ are perfectly fine!! However you MUST simplify expressions such as $\sin(\pi/3)$.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please include all information about the u-substitutions or integration by parts choice(s) that you make.**

$\sin^2 x = \frac{1}{2}(1-\cos(2x))$	$\cos^2 x = \frac{1}{2}(1+\cos(2x))$	$\sin(2x) = 2\sin(x)\cos(x)$

Expression	Substitution	
$\sqrt{a^2-x^2}$	$x = a\sin\theta,$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2+x^2}$	$x = a\tan\theta,$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2-a^2}$	$x = a\sec\theta,$	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
total	100	

DO EXACTLY FIVE OF THE SIX PROBLEMS!! Do not do all six problems. Make it clear which problem you are not attempting.

Integrals

Evaluate the integrals. Show all work and include all information about substitutions and integration by parts choices, and restrictions on angles for substitutions etc.

1. $\displaystyle\int_0^{(\pi/3)^2} \sin(\sqrt{t})\ dt$

2. $\displaystyle\int \frac{x^2+2x+10}{(x+3)(x^2+4)}\,dx$

3. $\displaystyle\int \frac{\sqrt{x^2-4}}{x^4}\,dx$

4. Comparison Theorem for Improper Integrals Fill in the blank.

Suppose $f(x)$ and $g(x)$ are continuous functions and $0 \leq f(x) \leq g(x)$ for all $x \geq a$.

(i) If $\int_a^\infty$ _____dx is divergent, then $\int_a^\infty$ _____dx is also divergent.

(ii) If $\int_a^\infty$ _____dx is convergent, then $\int_a^\infty$ _____dx is also convergent.

Use the Comparison Theorem for Improper Integrals to determine whether the integral is convergent or divergent.

$$\int_1^\infty \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\, dx$$

5. Arc Length

SET UP, but DO NOT EVALUATE the integral which calculates the circumference of the circle of radius 5.

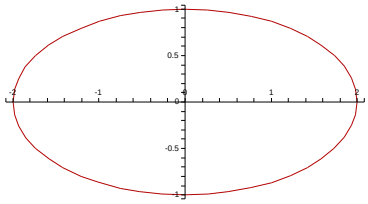
6. Parametric Curves. Graph matching. Write the number of the parametric graph that best matches the graph of the given parametric equation.

____ $\begin{cases} x = \cos(t) \\ y = 2\sin(t) \end{cases}$

____ $\begin{cases} x = \cos(2t) \\ y = \sin(2t) \end{cases}$

____ $\begin{cases} x = 1 - t \\ y = 2 + t \end{cases}$

____ $\begin{cases} x = 1 - t^2 \\ y = 2 + t^2 \end{cases}$



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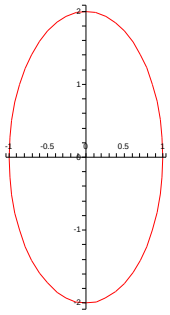

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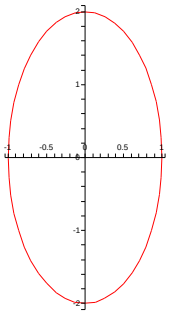

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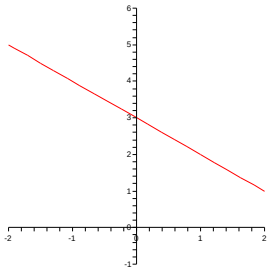

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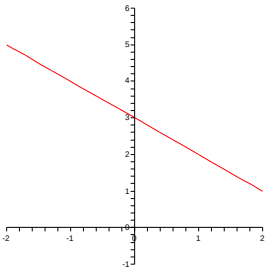

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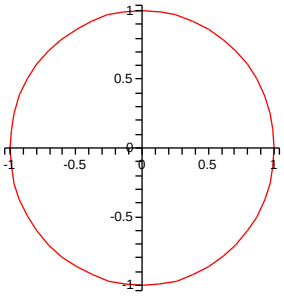

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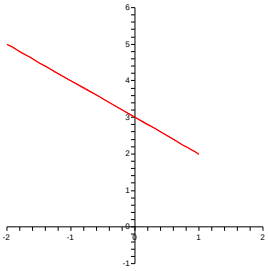

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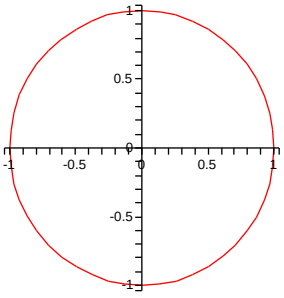

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