MATH 142 Midterm Exam #2

November 2, 2005 11:30AM

NAME:__________________________

- **No calculators are allowed on this exam.**
- **Answers such as** $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ **are perfectly fine!! However you MUST simplify expressions such as** $\sin(\pi/3)$**.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please include all information about the u-substitutions or integration by parts choice(s) that you make.**

$\sin^2 x = \frac{1}{2}(1-\cos(2x))$	$\cos^2 x = \frac{1}{2}(1+\cos(2x))$	$\sin(2x) = 2\sin(x)\cos(x)$

Expression	Substitution	
$\sqrt{a^2-x^2}$	$x = a\sin\theta$,	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$,	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$,	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

Problem	Points	Score
1	51	
2	16	
3	12	
4	6	
5	15	
total	100	

1. Integrals [17 points each]

DO EXACTLY THREE OF THE FOUR INTEGRALS BELOW!! Do not do all four problems. Make it clear which problem you are not attempting.

Evaluate the integrals. Show all work and include all information about substitutions and integration by parts choices, and restrictions on angles for substitutions etc. Simplify all trig expressions.

(a) $\displaystyle\int \frac{2x-13}{2x^2-5x-3}\,dx$

(b) $\displaystyle\int_0^1 \tan^{-1}(x)\,dx$

(c) $\int_0^{\pi/3} \cos^5(x)\tan^3(x)\ dx$

(d) $\int \frac{1}{(16+9x^2)^{3/2}}\ dx$

[16 points]

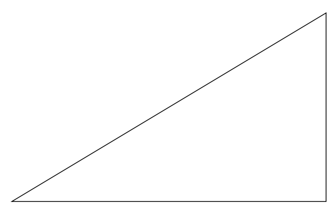
2. The sine function is not one-to-one, so its domain is restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ in order to define the inverse sine function.

$\sin^{-1}(x) = y$	means	$\sin(y) = x$ and, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Follow the steps below to find the derivative of the inverse sine function $\dfrac{d}{dx}(\sin^{-1}(x))$.
Note: You will not earn full credit for writing down the formula for the derivative of the inverse sine function. You should explain where the formula comes from by following the steps below.

(a) Use the definition of the inverse sine function, $\sin(y) = x$, to find a formula for $\frac{dy}{dx}$.

(b) Label all three sides of the right triangle to reflect the fact that $\sin(y) = x = \frac{x}{1}$.



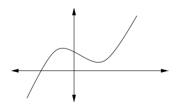
(c) Use the triangle above to find the formula for $\frac{dy}{dx}$ in terms of x.

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{dy}{dx} =$$

(d) Calculate $\frac{d}{dx}\left(\sin^{-1}(9x^2 + 2)\right)$

3. True or False. In the blank provided, write TRUE if the statement is always true or FALSE if the statement is sometimes false. [3 points each]

____________ The function $y = g(x)$ given below has an inverse function.



____________ If $f(x)$ is a one-to-one function with domain the interval $[2, 7]$ and range the interval $[-1, 4]$, then the inverse function has domain $[-1, 2]$ and range $[4, 7]$.

____________ The quantity $\log_5\left(\frac{1}{\sqrt{5}}\right)$ can be simplified to -2.

____________ The solution to the equation $2^x = 5$ is $\log_5(2)$.

4. Evaluate the following quantities: [6 points]

$\cos^{-1}(\cos(-\pi)) =$

$\sin^{-1}(\frac{1}{2}) =$

5. Find the inverse function of $f(x) = \dfrac{2x-1}{x+3}$. [15 points]